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How much time is needed to take an average eighth grade pupil from accuracy to 100% accuracy in a fundamental process of arithmetic --- Multiplication for example.

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Thesis

How Much Time is Needed to Take an Average  
Fifth or Sixth Grade Pupil from Inaccuracy  
to 100 % Accuracy in a Fundamental Process  
of Arithmetic, -- Multiplication for Example

Submitted by

Anne Josephine Caton

B.S. in Ed. Boston University 1934

In partial fulfillment of the requirements  
for the degree of Master of Education.

1936

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# THEORY OF THE EARTH

BY  
J. H. VAN DER KAM

LECTURER IN GEOL.

UNIVERSITY OF TORONTO

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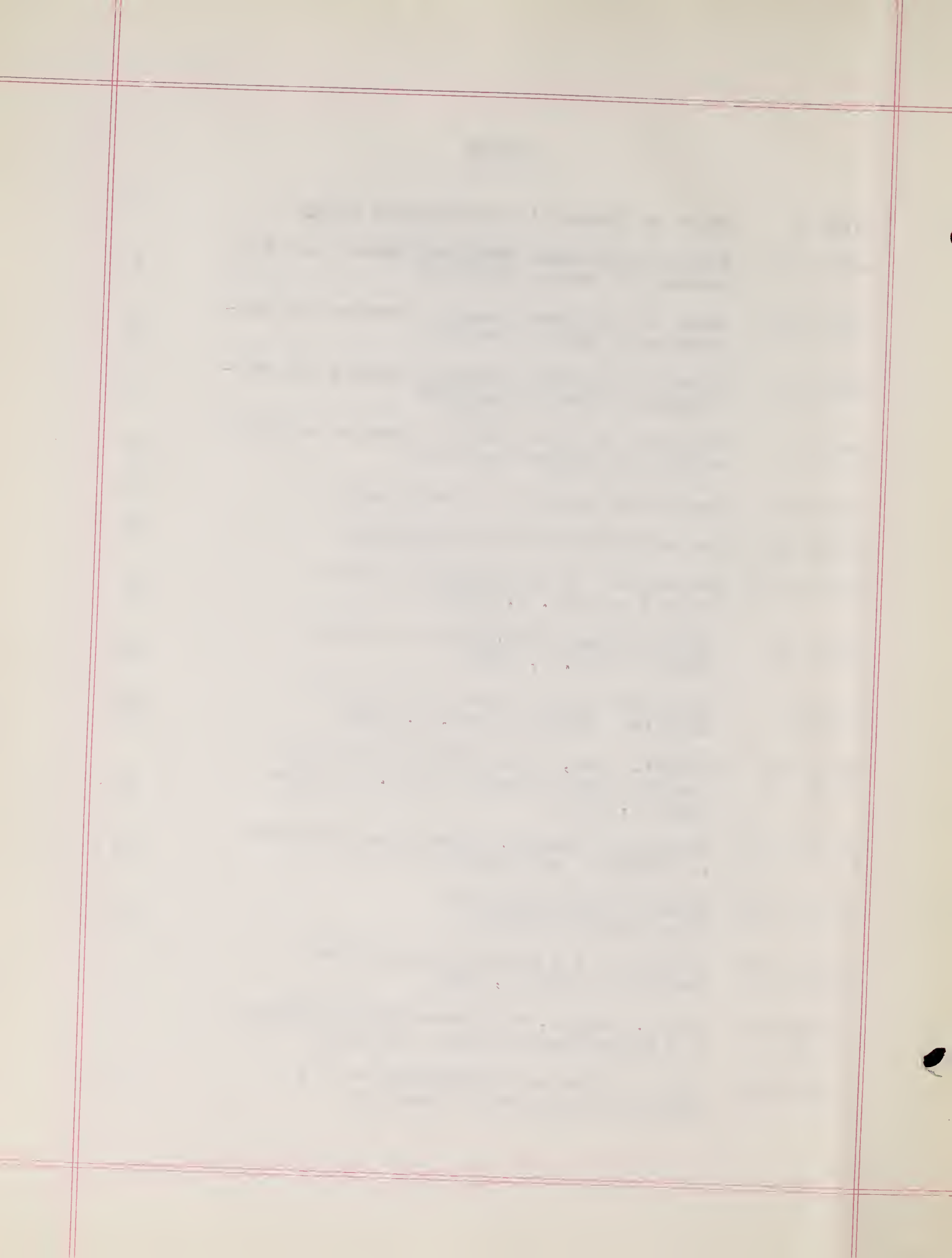
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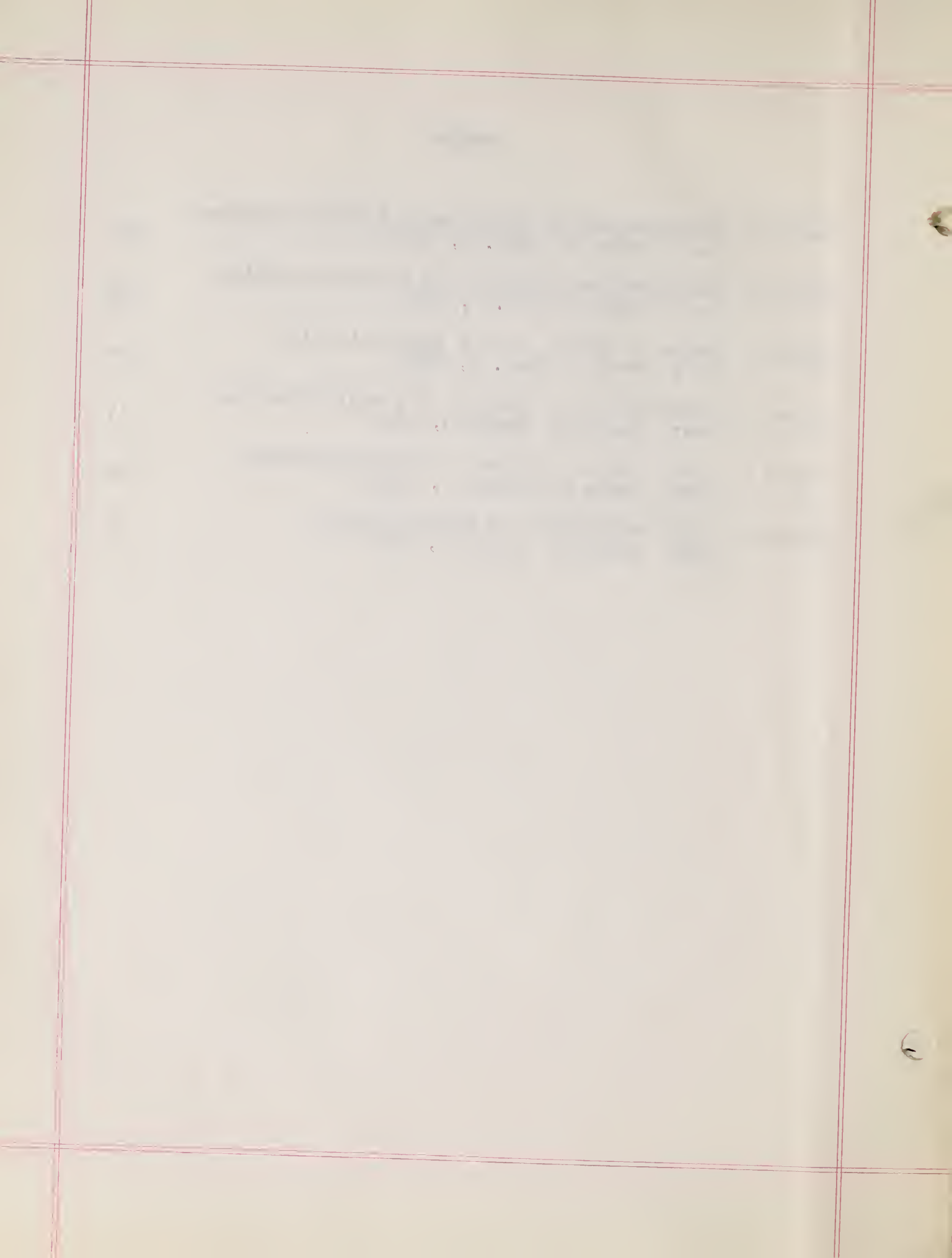


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## GRAPHS

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HOW MUCH TIME IS NEEDED TO TAKE AN AVERAGE FIFTH  
OR SIXTH GRADE PUPIL FROM INACCURACY TO 100%  
ACCURACY IN A FUNDAMENTAL PROCESS OF ARITHMETIC,  
MULTIPLICATION, FOR EXAMPLE?

## Chapter I. Introduction

### Problem:

How Much Time is Needed to Take an Average Fifth  
or Sixth Grade Pupil from Inaccuracy to 100% Accuracy in a  
Fundamental Process of Arithmetic, -Multiplication, for  
Example?

### Method.

An attempt will be made to diagnose types of errors  
in multiplication and to furnish remedial measures necessary  
to attain 100% accuracy in one fifth and one sixth grade.  
Part of the study will be devoted to the "Time" element from  
the standpoint of accuracy and not from the standpoint of  
speed. If some children are given time enough they might do  
their work accurately by reverting to the counting habit or  
"saying the tables." Gary Cleveland Myers refers to this  
method as the "pernicious counting habit." Even then we find  
"many inaccuracies. Automatic response to the multiplication  
combinations is the most necessary factor in attaining perfect  
mastery. Increase in time, therefore, becomes a danger signal  
and indicates the need of specific checking on habits of work.

In order to secure this the Wilson 5 P Test in Mul-  
tiplication was given to one fifth grade class and one sixth  
grade class of average ability. Specific remedial work was

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given to those pupils who did not receive a perfect score. Their "Time" was noted for habits of work. Twenty-minute periods of specific remedial work were given. The pupils were tested at intervals, the time required to do the test was recorded, and the number of periods of specific remedial work necessary for the pupil to obtain a perfect score were noted. Each pupil receiving a perfect score on a re-test was excused from further drill. The "Time" recorded included checking in all cases.

### Historical Background

Many changes in the teaching of arithmetic have occurred since 1900. Before that time, arithmetic, with all of its difficulties, was taken for granted. The more difficult the problem or the more intricate the process, the more discipline it gave for later life; such was the theory.

Arithmetic was placed in the curriculum in this country as early as 1750 because of the necessity of teaching computation in the development of industry. Gradually the time devoted to arithmetic was extended. During the first half of the next century the boys in many rural and small town schools devoted fully half of their time to arithmetic.

In an unpublished thesis entitled "Five Case Studies of Arithmetic Failure" Boston University School of Education, 1935, by R.F. Pucko, (36) the historical development of arithmetic is summarized. Further comments would be repetition.

(36) See Bibliography.



THESE ARE THE FIRST TWO VOLUMES OF THE HISTORY OF THE  
CITY OF NEW YORK, FROM THE FIRST SETTLEMENT TO THE  
PRESENT TIME. THE FIRST VOLUME CONTAINS THE HISTORY  
FROM THE FIRST SETTLEMENT TO THE YEAR 1789. THE  
SECOND VOLUME CONTAINS THE HISTORY FROM THE YEAR  
1789 TO THE PRESENT TIME. THE HISTORY IS  
WRITTEN IN A CLEAR AND CONCISE MANNER, AND  
IS ADAPTED TO THE USE OF STUDENTS AND  
OF THE GENERAL READER.

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Preliminary Study.

A preliminary study of errors in multiplication was made in February, 1934. Dr. Wilson's 5 P Multiplication test was given in a city system in Grades, five, six and seven in February, 1934, and in Grade four in May, 1934. Twenty-six types or errors were noted and tabulated.

One error in each incorrect example was found in Grade Four due to the fact that the work in multiplication was "fresh" in their minds. In Grades Five and Six some of the incorrect examples contained more than one error due probably to carelessness as the errors were inconsistent. In Grade Seven each incorrect example contained one error.

Total number of pupils tested			735
( 2 classes)	Grade IV	65	
(2 $\frac{1}{2}$ classes)	Grade V	93	
(2 $\frac{1}{2}$ classes)	Grade VI	100	
( 14 classes)	Grade VII	<u>477</u>	
			735

No. of examples		:	No. of incorrect examples		:	No. of errors	
		:			:		
IV	1625	:		477	:		477
V	2325	:		533	:		570
VI	2500	:		455	:		513
VII	11925	:		1509	:		1509

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The following is a list of the names of the members of the Society who have been elected to the office of President for the year 1900. The names are arranged in alphabetical order of their surnames. The names of the members who have been elected to the office of President for the year 1900 are: Sir John Lubbock, Bart., M.P.; Sir William Osler, Bart., M.D.; Sir Thomas Storer, Bart., M.D.; Sir James Spence, Bart., M.D.; Sir John Hall-Edwards, Bart., M.D.; Sir John Macleod, Bart., M.D.; Sir John Macpherson, Bart., M.D.; Sir John Macpherson, Bart., M.D.; Sir John Macpherson, Bart., M.D.; Sir John Macpherson, Bart., M.D.

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Sir William Osler, Bart., M.D.	10, Grosvenor Gardens	London
Sir Thomas Storer, Bart., M.D.	10, Grosvenor Gardens	London
Sir James Spence, Bart., M.D.	10, Grosvenor Gardens	London
Sir John Hall-Edwards, Bart., M.D.	10, Grosvenor Gardens	London
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Sir John Macpherson, Bart., M.D.	10, Grosvenor Gardens	London
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Table 1 on page 5 provides a summary and a comparison of percentage of errors in Grades Four to Seven inclusive, in this preliminary study. "Carrying in multiplication" received the highest percentage of errors in all grades. For the four grades combined it equalled 20.85%.

"Omitted one product" received the second highest percentage of errors in Grade Four--12.99%. In Grade Five the second highest percentage of errors occurred in "adding the partial products."--16.31%, while in Grade Six the second highest percentage of errors was found when "the first figure or the first two figures in the multiplier were a zero or zeros"--17.73%. In Grade Seven the second highest percentage of errors occurred, as in Grade Six, when "the first figure or the first two figures in the multiplier were a zero or zeros"--19.54%.

"Errors in adding the partial products" ranked third in Grades Four, Six, and Seven; Grade Four--12.15%; Grade Six--17.53%; Grade Seven--12.65%. In Grade Five "the first figure or the first two figures in the multiplier a zero or zeros" ranked third--15.26%.

"Multiplied incorrectly--no carrying" came fourth in Grades Five, Six, and Seven; Grade Five--8.94%; Grade Six--7.21%; Grade Seven--12.37%. The fourth percentage of errors in Grade Four occurred when "the first figure or the first two figures of the multiplier were a zero or zeros"--12.15%.





Number Five in percentage of errors in Grade Four occurred in "multiplying incorrectly--no carrying"--8.38%; in Grade Five "omitted one partial product"--8.59%; in Grade Six "put in a decimal point"--5.06%; in Grade Seven "omitted a dollars sign"--3.84%.

Number Six in percentage of errors in Grade Four--"the partial product in the wrong place"--6.70%; in Grade Five--"put in a decimal point"--4.83%; Grade Six--"zero times digit"--4.67%; Grade Seven "multiplied the partial products instead of adding them"--3.65%.

Number seven in percentage of errors in Grade Four--"multiplied the partial products instead of adding them"--5.03%; Grade Five "omitted the dollars sign"--3.33%; Grade Six--"omitted the dollars sign"--4.09%; Grade seven--"omitted one partial product"--3.04%.

Number eight in percentage of errors in Grade Four--"digit times zero"--3.14%; Grade Five--"zero times digit"--3.15%; Grade six--"digit times zero"--4.09%; Grade Seven--"Zero times digit"--2.31%.

It is evident from Table 1 that carrying, zero difficulties, and adding partial products are major causes of difficulty in multiplication. Together they account for 49.98% of the errors of the 735 pupils in Grades Four, Five, Six, and Seven who took the Wilson 5 P Multiplication Test in February and May, 1934. In Tables II, III, IV, and V, respectively, the eight most frequent errors for each grade are shown.





In each grade carrying stands at the top of the list, as a cause of error.

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Table 1

5

Showing the Types of Errors in the Preliminary Study  
in Grades Four, Five, Six, and Seven, as shown by  
the returns in the 5 P Multiplication Test Feb,  
12, 1934; Feb. 19, 1934; May 19, 1934

	Gr. IV	Gr. V	Gr. VI	Gr. VII
	65	93	100	477
1 Multiplied wrong-				
no carrying-----	40	51	37	187
2 Carrying in multiplication:	120	112	136	472
3 Added instead of				
multiplying-----	1	10	2	26
4 Errors in adding products--:	58	93	90	191
5 Did not add the products--:	7	6	2	6
6 Dollars and cents				
times ten-----	12	16	11	11
7 Omitted dollars sign-----:	0	19	21	58
8 Omitted decimal point-----:	0	8	8	10
9 Omitted dollars sign and				
decimal point in same				
example	6	4	10	31
10 Put in a decimal point----	5	25	26	8
11 Decimal point in				
wrong place-----	1	5	4	16
12 Zero times digit-----	7	18	24	35
13 Digit times zero-----	15	9	21	21
14 Zero in multiplicand-----:	5	12	3	9
15 Zero in multiplicand and				
multiplier-----	0	2	1	7
16 Omitted one product-----:	62	49	14	46
17 Did not multiply last				
figure in multiplicand---	2	3	0	3
18 Multiplied last figure in :				
multiplicand twice-----	2	1	2	0
19 Put in extra figure in the:				
answer or in the products:	6	2	4	2
20 Left out one figure in the:				
product 9 x 4--3(6)				
omitting the 6-----	10	3	1	1
21 Omitted the example-----:	1	11	0	11
22 Began with second figure				
in multiplicand for the				
second product-----	2	8	0	1
23 Product in the wrong				
place-----	32	13	4	2
24 Errow when first figure or:				
first two figures in mul-				
tiplier-zero or zeros----	58	87	91	295

The following table shows the results of the survey conducted in the year 1921, and is intended to give a general idea of the conditions prevailing in the various districts of the country. The figures are given in thousands of pounds sterling, and are rounded off to the nearest thousand.

District					Total	Remarks
Area	Population	Value of Land	Value of Buildings	Value of Stock		
1	100	100	100	100	400	
2	200	200	200	200	800	
3	300	300	300	300	1200	
4	400	400	400	400	1600	
5	500	500	500	500	2000	
6	600	600	600	600	2400	
7	700	700	700	700	2800	
8	800	800	800	800	3200	
9	900	900	900	900	3600	
10	1000	1000	1000	1000	4000	
11	1100	1100	1100	1100	4400	
12	1200	1200	1200	1200	4800	
13	1300	1300	1300	1300	5200	
14	1400	1400	1400	1400	5600	
15	1500	1500	1500	1500	6000	
16	1600	1600	1600	1600	6400	
17	1700	1700	1700	1700	6800	
18	1800	1800	1800	1800	7200	
19	1900	1900	1900	1900	7600	
20	2000	2000	2000	2000	8000	
21	2100	2100	2100	2100	8400	
22	2200	2200	2200	2200	8800	
23	2300	2300	2300	2300	9200	
24	2400	2400	2400	2400	9600	
25	2500	2500	2500	2500	10000	
26	2600	2600	2600	2600	10400	
27	2700	2700	2700	2700	10800	
28	2800	2800	2800	2800	11200	
29	2900	2900	2900	2900	11600	
30	3000	3000	3000	3000	12000	
31	3100	3100	3100	3100	12400	
32	3200	3200	3200	3200	12800	
33	3300	3300	3300	3300	13200	
34	3400	3400	3400	3400	13600	
35	3500	3500	3500	3500	14000	
36	3600	3600	3600	3600	14400	
37	3700	3700	3700	3700	14800	
38	3800	3800	3800	3800	15200	
39	3900	3900	3900	3900	15600	
40	4000	4000	4000	4000	16000	
41	4100	4100	4100	4100	16400	
42	4200	4200	4200	4200	16800	
43	4300	4300	4300	4300	17200	
44	4400	4400	4400	4400	17600	
45	4500	4500	4500	4500	18000	
46	4600	4600	4600	4600	18400	
47	4700	4700	4700	4700	18800	
48	4800	4800	4800	4800	19200	
49	4900	4900	4900	4900	19600	
50	5000	5000	5000	5000	20000	

Table 1 (continued)

	Gr. IV	Gr. V	Gr. VI	Gr. VII
	65	93	100	477
25 Multiplied instead of adding the products---	24	3	0	55
26 Omitted a figure in the: multiplicand not other- wise covered-----	1	0	2	5
Total number of errors--	477	570	513	1509





Showing Eight of the Most Frequent Errors and the Percentage of Errors in Grade Four as shown by the Analysis of the 5P Multiplication Test, May 17, 1934

1	Carrying in multiplication	120	25.15 %
2	Omitted one product	62	12.99 %
3	Errors in adding the products	58	12.15 %
4	Errors when first figure or first two figures in multiplier--zero or zeros	58	12.15 %
5	Multiplied incorrectly--no carrying	40	8.38 %
6	Product in the wrong place	32	6.70 %
7	Multiplied instead of adding the products	24	5.03 %
8	Digit times Zero	15	3.14 %
Total of the eight most frequent errors		409	
Total number of errors		477	
Total number of pupils (two classes)		65	

The above table is read as follows:

Carrying in multiplication ranked first in the types of errors.

There were 120 errors of this type.

Tables III, IV, and V are read in the same manner.



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1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9
10	10	10	10

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Showing Eight of the Most Frequent Errors and  
the Percentage of Errors in Grade Five as  
shown by the Analysis of the 5 P Multipli-  
cation Test, Feb. 12, 1934

1	Carrying in multiplication	112	19.64 %
2	Errors in adding the products	93	16.31 %
3	Error when first figure or first two figures in multiplier-zero or zeros	87	15.26 %
4	Multiplied incorrectly-no carrying	51	8.94 %
5	Omitted one product	49	8.59 %
6	Put in a decimal point	25	4.38%
7	Omitted dollars sign	19	3.33 %
8	Zero times digit	18	3.15 %
Total of the eight most frequent errors		454	79.64 %
Total number of errors		570	
Total number of pupils ( $2\frac{1}{2}$ classes)		93	

IN SENATE,  
January 10, 1906.  
REPORT OF THE  
COMMISSIONERS OF THE LAND OFFICE.

1. 1894	10	1. 1894	10	1. 1894	10
2. 1895	11	2. 1895	11	2. 1895	11
3. 1896	12	3. 1896	12	3. 1896	12
4. 1897	13	4. 1897	13	4. 1897	13
5. 1898	14	5. 1898	14	5. 1898	14
6. 1899	15	6. 1899	15	6. 1899	15
7. 1900	16	7. 1900	16	7. 1900	16
8. 1901	17	8. 1901	17	8. 1901	17
9. 1902	18	9. 1902	18	9. 1902	18
10. 1903	19	10. 1903	19	10. 1903	19
11. 1904	20	11. 1904	20	11. 1904	20
12. 1905	21	12. 1905	21	12. 1905	21
13. 1906	22	13. 1906	22	13. 1906	22
14. 1907	23	14. 1907	23	14. 1907	23
15. 1908	24	15. 1908	24	15. 1908	24
16. 1909	25	16. 1909	25	16. 1909	25
17. 1910	26	17. 1910	26	17. 1910	26
18. 1911	27	18. 1911	27	18. 1911	27
19. 1912	28	19. 1912	28	19. 1912	28
20. 1913	29	20. 1913	29	20. 1913	29
21. 1914	30	21. 1914	30	21. 1914	30
22. 1915	31	22. 1915	31	22. 1915	31
23. 1916	32	23. 1916	32	23. 1916	32
24. 1917	33	24. 1917	33	24. 1917	33
25. 1918	34	25. 1918	34	25. 1918	34
26. 1919	35	26. 1919	35	26. 1919	35
27. 1920	36	27. 1920	36	27. 1920	36
28. 1921	37	28. 1921	37	28. 1921	37
29. 1922	38	29. 1922	38	29. 1922	38
30. 1923	39	30. 1923	39	30. 1923	39
31. 1924	40	31. 1924	40	31. 1924	40
32. 1925	41	32. 1925	41	32. 1925	41
33. 1926	42	33. 1926	42	33. 1926	42
34. 1927	43	34. 1927	43	34. 1927	43
35. 1928	44	35. 1928	44	35. 1928	44
36. 1929	45	36. 1929	45	36. 1929	45
37. 1930	46	37. 1930	46	37. 1930	46
38. 1931	47	38. 1931	47	38. 1931	47
39. 1932	48	39. 1932	48	39. 1932	48
40. 1933	49	40. 1933	49	40. 1933	49
41. 1934	50	41. 1934	50	41. 1934	50
42. 1935	51	42. 1935	51	42. 1935	51
43. 1936	52	43. 1936	52	43. 1936	52
44. 1937	53	44. 1937	53	44. 1937	53
45. 1938	54	45. 1938	54	45. 1938	54
46. 1939	55	46. 1939	55	46. 1939	55
47. 1940	56	47. 1940	56	47. 1940	56
48. 1941	57	48. 1941	57	48. 1941	57
49. 1942	58	49. 1942	58	49. 1942	58
50. 1943	59	50. 1943	59	50. 1943	59
51. 1944	60	51. 1944	60	51. 1944	60
52. 1945	61	52. 1945	61	52. 1945	61
53. 1946	62	53. 1946	62	53. 1946	62
54. 1947	63	54. 1947	63	54. 1947	63
55. 1948	64	55. 1948	64	55. 1948	64
56. 1949	65	56. 1949	65	56. 1949	65
57. 1950	66	57. 1950	66	57. 1950	66
58. 1951	67	58. 1951	67	58. 1951	67
59. 1952	68	59. 1952	68	59. 1952	68
60. 1953	69	60. 1953	69	60. 1953	69
61. 1954	70	61. 1954	70	61. 1954	70
62. 1955	71	62. 1955	71	62. 1955	71
63. 1956	72	63. 1956	72	63. 1956	72
64. 1957	73	64. 1957	73	64. 1957	73
65. 1958	74	65. 1958	74	65. 1958	74
66. 1959	75	66. 1959	75	66. 1959	75
67. 1960	76	67. 1960	76	67. 1960	76
68. 1961	77	68. 1961	77	68. 1961	77
69. 1962	78	69. 1962	78	69. 1962	78
70. 1963	79	70. 1963	79	70. 1963	79
71. 1964	80	71. 1964	80	71. 1964	80
72. 1965	81	72. 1965	81	72. 1965	81
73. 1966	82	73. 1966	82	73. 1966	82
74. 1967	83	74. 1967	83	74. 1967	83
75. 1968	84	75. 1968	84	75. 1968	84
76. 1969	85	76. 1969	85	76. 1969	85
77. 1970	86	77. 1970	86	77. 1970	86
78. 1971	87	78. 1971	87	78. 1971	87
79. 1972	88	79. 1972	88	79. 1972	88
80. 1973	89	80. 1973	89	80. 1973	89
81. 1974	90	81. 1974	90	81. 1974	90
82. 1975	91	82. 1975	91	82. 1975	91
83. 1976	92	83. 1976	92	83. 1976	92
84. 1977	93	84. 1977	93	84. 1977	93
85. 1978	94	85. 1978	94	85. 1978	94
86. 1979	95	86. 1979	95	86. 1979	95
87. 1980	96	87. 1980	96	87. 1980	96
88. 1981	97	88. 1981	97	88. 1981	97
89. 1982	98	89. 1982	98	89. 1982	98
90. 1983	99	90. 1983	99	90. 1983	99
91. 1984	100	91. 1984	100	91. 1984	100
92. 1985	101	92. 1985	101	92. 1985	101
93. 1986	102	93. 1986	102	93. 1986	102
94. 1987	103	94. 1987	103	94. 1987	103
95. 1988	104	95. 1988	104	95. 1988	104
96. 1989	105	96. 1989	105	96. 1989	105
97. 1990	106	97. 1990	106	97. 1990	106
98. 1991	107	98. 1991	107	98. 1991	107
99. 1992	108	99. 1992	108	99. 1992	108
100. 1993	109	100. 1993	109	100. 1993	109

Showing Eight of the Most Frequent Errors and  
the Percentage of Errors in Grade Six as  
shown by the Analysis of the 5 P Multipli-  
cation Test. Feb. 12, 1934

1	Carrying in multiplication	136	26.51 %
2	Error when first figure or first two figures in multiplier-zero or zeros	91	17.73 %
3	Errors in adding the product	90	17.54 %
4	Multiplied incorrectly--no carrying	37	7.21 %
5	Put in a decimal point	26	5.06 %
6	Zero times digit	24	4.67 %
7	Omitted dollars sign	21	4.09 %
8	Digit times zero	21	4.09 %
Total of the eight most frequent errors		446	86.93 %
Total number of errors		513	
Total number of pupils ( $2\frac{1}{2}$ classes)		100	

the whole business was not so much as  
the other part of the business and  
the other part of the business was  
not so much as the other part of the business

1. 1. 10	10	the whole business was not so much as	1
2. 1. 10	20	the other part of the business and	2
3. 1. 10	30	the other part of the business was	3
4. 1. 10	40	the other part of the business was	4
5. 1. 10	50	the other part of the business was	5
6. 1. 10	60	the other part of the business was	6
7. 1. 10	70	the other part of the business was	7
8. 1. 10	80	the other part of the business was	8
9. 1. 10	90	the other part of the business was	9
10. 1. 10	100	the other part of the business was	10
11. 1. 10	110	the other part of the business was	11
12. 1. 10	120	the other part of the business was	12
13. 1. 10	130	the other part of the business was	13
14. 1. 10	140	the other part of the business was	14
15. 1. 10	150	the other part of the business was	15
16. 1. 10	160	the other part of the business was	16
17. 1. 10	170	the other part of the business was	17
18. 1. 10	180	the other part of the business was	18
19. 1. 10	190	the other part of the business was	19
20. 1. 10	200	the other part of the business was	20

Table V

Showing Eight of the Most Frequent Errors and  
the Percentage of Errors in Grade Seven as  
shown by the Analysis of the 5 P Multipli-  
cation Test. Feb. 19, 1934

1	Carrying in multiplication	472	31.27 %
2	Error when first figure or first two figures in multiplier-zero or zeros	295	19.54 %
3	Errors in adding the products	191	12.65 %
4	Multiplied incorrectly--no carrying	187	12.39 %
5	Omitted dollars sign	58	3.84 %
6	Multiplied instead of adding the products	55	3.65 %
7	Omitted one product	46	3.04 %
8	Zero times digit	35	2.31 %
Total of the eight most frequent errors		1339	88.73 %
Total number of errors		1509	
Total number of pupils ( 14 classes )		477	







In Grade Four there were two perfect scores; in Grade Five there was one perfect score; in Grade Six there was one perfect score; and in Grade seven there were seventy-three perfect scores. In the four grades there were seventy-seven perfect scores equalling 10.99%.

In Table VI will be found the summary of the scores for each of the four grades, and in Table VII will be found the summary of the median, quartile three, quartile one, high score, low score, and standard deviation for each of the grades.



Showing Summary of the Scores for Each Grade,  
in the Preliminary Study in the Spring of  
1934

<u>Score</u>	:	<u>Grade Four</u>	:	<u>Grade Five</u>	:	<u>Grade Six</u>	:	<u>Grade Seven</u>
100	:	2	:	1	:	1	:	73
96	:	5	:	2	:	14	:	69
92	:	3	:	8	:	12	:	68
88	:	7	:	16	:	18	:	85
84	:	4	:	15	:	14	:	55
80	:	13	:	8	:	10	:	35
76	:	3	:	13	:	9	:	46
72	:	8	:	8	:	8	:	26
68	:	3	:	8	:	4	:	6
64	:	2	:	2	:	5	:	5
60	:	2	:	2	:	1	:	5
56	:	1	:	3	:	0	:	1
52	:	1	:	3	:	0	:	1
48	:	2	:	1	:	1	:	1
44	:	3	:	1	:	1	:	0
40	:	0	:	1	:	0	:	1
36	:	1	:	0	:	1	:	0
32	:	1	:	0	:	1	:	0
28	:	1	:	0	:	0	:	0
24	:	1	:	0	:	0	:	0
20	:	1	:	1	:	0	:	0
16	:	1	:	0	:	0	:	0

The percentage of perfect scores in the four grades was 10.47 %.

The above table is read as follows:

Two pupils in Grade Four, one pupil in Grade Five, one pupil in Grade Six, and seventy-three pupils in Grade Seven received perfect scores.



A Point Summary of the Scores for Each Grade  
in the Preliminary Study in the Spring of  
1934

	<u>High</u>	<u>Q-3</u>	<u>Median</u>	<u>Q-1</u>	<u>Low</u>	<u>S.D.</u>
Grade 4	100	88	80	64	16	21.30
Grade 5	100	88	80	72	20	13.52
Grade 6	100	92	84	76	32	12.63
Grade 7	100	92	88	80	40	9.96

The above table is read as follows:

In Grade Four the median score was 80; third quartile 88; first quartile 64; high score 100; low score 16; standard deviation 21.30.





## Chapter III

The Zero Difficulty

The "zero difficulty" is discussed briefly here as an illustration of the difficulties which confront us in multiplication. Some children have no difficulty with the digit times zero but do have difficulty with the zero times the digit. On the other hand some children have no difficulty with the zero times the digit but do have trouble with the digit times the zero. Others have difficulty with both.

The "zero difficulty" and "carrying in multiplication" are the two types of errors most commonly reported upon by workers in research on multiplication difficulties. Here are three comments by research students:

"The zero is one of the greatest arithmetic trouble makers in the elementary school." (22)

"Children who are learning arithmetic must actually employ the zero for what it is intended--a place holder." (44)

"The most common fault in multiplication is lack of knowledge of the basic combinations especially those in which the zero is involved. This fault is also revealed by the pupils who count or repeat the tables to procure the product.

Another source of error in multiplication is faulty carrying, thereby revealing a weakness in addition.

Numerous faulty procedures are found in examples involving the work of multiplying by two or more figures especially when the multiplier contains one or more zeros.

The analysis of the data concerning faulty procedures suggests the following:

(1) The necessity of giving special attention to examples involving zeros in either multiplicand or multiplier.

(2) The necessity of insisting on neat work and the correct placing of partial products.

(3) The desirability of requiring the pupil to re-check the entire work before being prepared to accept the answer as correct." (7)

CHAPTER 10

The first thing I noticed when I stepped out of the car was the cold. It was a sharp contrast to the warm blanket I had been sitting under. I looked around, trying to get my bearings. The street was empty, the only sound being the distant hum of traffic. I felt a little disoriented, but I knew I had to keep moving. I started walking, my feet hitting the cold pavement. The air was crisp, and I could feel my breath fogging in front of me. I was alone, and it felt like the world had stopped for a moment. I kept walking, not knowing where I was going, but feeling a sense of purpose. The cold was a wake-up call, and I was ready to answer it.

I walked for what felt like hours, the cold never letting up. My body was starting to numb, but I didn't stop. I was determined to find my way out of this place. The streetlights were dim, and the shadows were long. I felt like I was being watched, but I didn't look back. I was focused on the path ahead of me, and I knew I was getting closer to the end of the road.

Just as I was about to give up, I saw a light in the distance. It was a small, warm glow that seemed to be calling to me. I started running, my heart pounding in my chest. I knew that light was my chance, and I was not going to let it slip away. I ran faster, my legs pumping, and I felt a surge of energy. I was finally going to make it.

I reached the light, and it was a doorway to another world. The cold disappeared, and I was surrounded by warmth and light. I took a deep breath, and I felt like I had been reborn. I was no longer lost, and I was no longer alone. I was home. I looked around, and I saw a group of people standing there, watching me. They were all smiling, and I knew that I had done it. I had found my way out, and I was free. I felt a sense of accomplishment, and I knew that I was ready for whatever came next. I was no longer a stranger in a strange land, and I was no longer afraid. I was a survivor, and I was proud of what I had achieved.

I walked away from the light, feeling a sense of peace and contentment. I knew that I had found my purpose, and I was ready to face the world. I was no longer a victim, and I was no longer a stranger. I was a person, and I was proud of who I was. I was a survivor, and I was ready for whatever came next. I was no longer a stranger in a strange land, and I was no longer afraid. I was a survivor, and I was proud of what I had achieved.

There is practically no variation in the types of errors in multiplication in the different research studies which were examined by the writer. During the first semester of 1924-1925 a test containing 210 examples in multiplication was given to two thousand pupils in Grades Four B to Eight A inclusive in twelve schools in the city of Milwaukee, Wisconsin. The findings were as follows:

"(1) Specific drill during special help time for two weeks reduced the errors from 7.6 in the first test to 0.8 in the third test.

(2) There proved to be several 'zero' difficulties instead of one as is commonly accepted. The per cent of errors in examples with zeros in the units place was found to be almost always greater than in examples with zeros in the tens place.

(3) Certain types of errors especially where the zero occurred in the units place persisted even though specific drill was given.

(4) The greatest source of error in carrying occurred when the sum of the carried number and the partial product was in a decade above the partial product as in the following example:

$$\begin{array}{r} 888 \\ \times 7 \\ \hline 7216 \end{array} \quad \text{16 added instead of 6}$$

(5) Drill upon specific skills in which pupils were weak yielded larger returns than indiscriminate drills.

(6) The diagnostic tests revealed the weak skills, while drill cards enabled the teachers to give specific drill upon the specific weaknesses of each pupil.

(7) That forgetting played an important part was proved by the fact that the per cent of error increased from 0.8 in test 3 to 1.4 in test 4. The time between test 3 and test 4 was three months, during which no specific drill was given." (12)







Drill

Since multiplication is so closely allied to addition it will be necessary for the pupil to meet the addition requirements:

"The 100 primary or first decade facts.

The 300 upper decade facts through 399.

The 80 additional higher facts needed for carrying in multiplication through  $9 \times 9$ ." (49)

These added to the 100 multiplication combinations through  $9 \times 9$  complete the requirements for 100 per cent mastery as far as the multiplication facts are concerned. Beyond the facts are the process difficulties, and these must be mastered also.

Can the school child be given complete control of the fundamentals in school life? This study tends to prove that it can if the load is properly reduced and the material well organized.

"More attention should be directed toward a definite attempt to secure 100 per cent accuracy in arithmetic. With correctly chosen subject matter and a well organized teaching program the average child becomes perfect in the ground work of the fundamentals.

Reasons for failure in the past:

1. More than half of the time has been spent on useless processes, processes never used by the adult community and therefore not likely to be used outside of the schoolroom by the pupils.

2. Beginning formal drill too early.

3. Absence of adequate teaching plans. There has been no plan for teaching facts in sizable units and there has been no plan for checking.

4. Lack of adequate drill service.

In the attempt to gain 100 per cent results in arithmetic the first step is the proper choice of subject matter. 95 per cent of all arithmetic used in all occupations consists of simple manipulations in the four fundamentals." (57)

1. The first part of the document discusses the importance of maintaining accurate records of all transactions. It emphasizes that proper record-keeping is essential for the transparency and accountability of the organization. This section also outlines the various methods used to collect and analyze data, ensuring that the information is reliable and up-to-date.

2. The second part of the document focuses on the financial aspects of the organization. It provides a detailed overview of the budget, including the projected income and expenses for the upcoming year. This section also discusses the various financial risks and how they can be mitigated, ensuring that the organization remains financially stable and secure.

3. The third part of the document addresses the human resources of the organization. It discusses the current staffing levels and the need for additional personnel in certain areas. This section also outlines the various training and development programs that are available to employees, ensuring that they are equipped with the skills and knowledge needed to perform their jobs effectively.

4. The fourth part of the document discusses the legal and regulatory requirements that the organization must comply with. It provides a detailed overview of the various laws and regulations that apply to the organization, ensuring that it remains in full compliance with all applicable laws. This section also discusses the various legal risks and how they can be managed, ensuring that the organization remains legally sound and secure.

5. The fifth part of the document discusses the overall performance of the organization. It provides a detailed overview of the various key performance indicators (KPIs) that are used to measure the organization's success. This section also discusses the various challenges that the organization is facing and how they can be overcome, ensuring that the organization remains competitive and successful in the long run.

The Twenty Ninth Yearbook of the National Society for the Study of Education gives the following aspects of modern thought on arithmetic:

"We drill to build a skill and we drill to maintain a skill.

Drill and application should follow effective learning of the content.

Instruction and drill should be interspersed.

The intention to learn should be dominant, not for the moment, but permanently.

Clear well-understood ideas of a process should be grasped before drilling upon it.

Careful construction of drill should be built so that certain combinations are not forever beggars to the child.

The work should be presented in such varieties of situations that the child is not lost the moment the problem is situated in a way to which he is not accustomed.

There should be genuine motivation of effort.

Careful testing should reveal the weak points followed by prudent re-teaching and drill before such weak points become chronic.

There should be a judicious grading of work to varying abilities." (24)

Drill cannot be effective unless the causes of the types of errors can be discovered and corrected by application of the particular instruction which the individual pupil needs. Such instruction will correct the erroneous methods of procedure; group instruction is not enough.

Greene and Buswell in their article on "Testing, Diagnosis, and Remedial Work" in the Twenty-Ninth Yearbook of the National Society for the Study of Education (18) make the following statements:





"Group diagnostic testing should always come first.

Individual diagnosis testing follows with the further analysis of the particular difficulties which the pupil experiences in order that his method of working may be made more effective.

The chief purpose of individual diagnosis is to discover how the pupil works when he proceeds in his regular manner.

Diagnostic procedure and remedial procedure should be separate processes and the teacher should not attempt to remedy a poor method while making a diagnosis.

Real teaching must be individual teaching to a very considerable extent.

Effective teaching is specific teaching.

Careful individual diagnosis must be made in the case of children who have serious difficulties.

It is a mistake to give all pupils the same drill. It is a waste of time for those who do not need the drill.

The basic principles underlying individual diagnostic and remedial work is that improvement in the results obtained will be sought by an improvement in the methods by which these results are obtained." (18)

Remedial work provides for individual differences.

It not only furnishes corrective work for the pupil who is slow but it also provides enrichment for the pupil who is able to proceed at a more rapid rate. It should also eliminate that which is not practical or useful in arithmetic. In this manner the load will be lightened and the joy of accomplishment can be experienced by every pupil of normal intelligence.

"Suit the amount of drill to the difficulty of the task.

Teach that which is most used in life.

Teach that which the pupils do not know.

Do not try to teach what the child could not learn even with the best teacher in the world.

Always do your best to encourage pupils to want to learn what they should learn.

Teach what is most used or should be most used in life." (32)



The first part of the report deals with the general situation of the country and the progress of the work during the year. It is followed by a detailed account of the various projects and the results achieved. The report concludes with a summary of the work done and a list of the names of the persons who have taken part in it.

The second part of the report deals with the financial situation of the country and the progress of the work during the year. It is followed by a detailed account of the various projects and the results achieved. The report concludes with a summary of the work done and a list of the names of the persons who have taken part in it.

The third part of the report deals with the financial situation of the country and the progress of the work during the year. It is followed by a detailed account of the various projects and the results achieved. The report concludes with a summary of the work done and a list of the names of the persons who have taken part in it.

In summary, the aims are about as follows:-

Drill material should be suited to the every day needs of the pupils and useless material eliminated.

Drill material should be suited to the individual needs of the pupils.

Drill must be specific.

Drill should be well organized.

Drill should follow learning.

Drill to build a skill.

Drill to maintain a skill.

Perfect mastery of the four fundamentals must come through understanding followed by automatic response.

1. The first part of the paper is devoted to a general discussion of the problem.

2. The second part is devoted to a detailed analysis of the case of a single particle.
3. The third part is devoted to a detailed analysis of the case of a system of particles.
4. The fourth part is devoted to a detailed analysis of the case of a system of particles.
5. The fifth part is devoted to a detailed analysis of the case of a system of particles.
6. The sixth part is devoted to a detailed analysis of the case of a system of particles.
7. The seventh part is devoted to a detailed analysis of the case of a system of particles.
8. The eighth part is devoted to a detailed analysis of the case of a system of particles.
9. The ninth part is devoted to a detailed analysis of the case of a system of particles.
10. The tenth part is devoted to a detailed analysis of the case of a system of particles.

Specific Remedial Work in Grade Five and the Results

From the Preliminary Study previously reported in Chapter 1 the writer reached the conclusion that any child of normal intelligence above the fourth grade can attain 100 per cent mastery in multiplication if the load is properly reduced and the material well organized.

On Nov. 5, 1935 the Wilson 5 P Multiplication Test was given to 38 fifth grade pupils ranging in mental ability from 67 I.Q. to 145 I.Q. The I.Q.'s were determined by the National Intelligence Tests given earlier in the year.

## Distribution of I.Q.'s

<u>No. of</u> <u>pupils</u>	<u>I.Q.</u>	<u>No. of</u> <u>pupils</u>	<u>I.Q.</u>
1-----	67	1-----	108
1-----	78	1-----	110
1-----	83	1-----	111
1-----	86	2-----	113
2-----	87	1-----	114
1-----	89	3-----	115
1-----	93	1-----	116
1-----	99	1-----	118
2-----	100	2-----	121
1-----	101	1-----	122
1-----	104	1-----	131
3-----	105	1-----	132
1-----	106	2-----	133
2-----	107	1-----	145
Total		38	

From the distribution of the I.Q.'s this does not seem like an average class. An explanation later in the study accounts for some of the low I.Q.'s.





Showing Scores made on the 5 P Multiplication Test  
by 38 Fifth Grade Pupils, Nov. 5, 1935.

<u>No. of pupils</u>	<u>Score</u>	
2-----	100	
3-----	96	
10-----	92	-----Q3-----92
4-----	88	
6-----	84	-----Median-----84
3-----	80	
3-----	76	-----Q1-----76
4-----	72	
1-----	68	
1-----	64	
1-----	28	
<u>38</u>		

Showing The Time taken by 38 pupils of the Fifth Grade  
on the 5 P Multiplication Test: Nov. 5, 1935.

<u>No. of pupils</u>	<u>Time</u>	
1-----	13	
6-----	18	
8-----	19	-----Q3-----19
2-----	20	
3-----	22	-----Median-----22
5-----	23	
2-----	24	
2-----	25	
2-----	27	-----Q1-----27
1-----	29	
1-----	30	
1-----	31	
1-----	32	
1-----	34	
1-----	37	
1-----	40	
<u>38</u>		

The above data on scores and time is shown graphically  
in Figures 1 and 2 which follow.

THE UNIVERSITY OF CHICAGO

NAME	ADDRESS
ALAN TURING	15, Avenue Road, Cambridge, England
ALAN TURING	15, Avenue Road, Cambridge, England
ALAN TURING	15, Avenue Road, Cambridge, England
ALAN TURING	15, Avenue Road, Cambridge, England
ALAN TURING	15, Avenue Road, Cambridge, England
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THE UNIVERSITY OF CHICAGO

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ALAN TURING	15, Avenue Road, Cambridge, England
ALAN TURING	15, Avenue Road, Cambridge, England
ALAN TURING	15, Avenue Road, Cambridge, England

THE UNIVERSITY OF CHICAGO

Figure 1

Showing graphically the distribution of scores on the  
5 P Multiplication Test made by 38 Fifth Grade pupils

Nov. 5, 1935

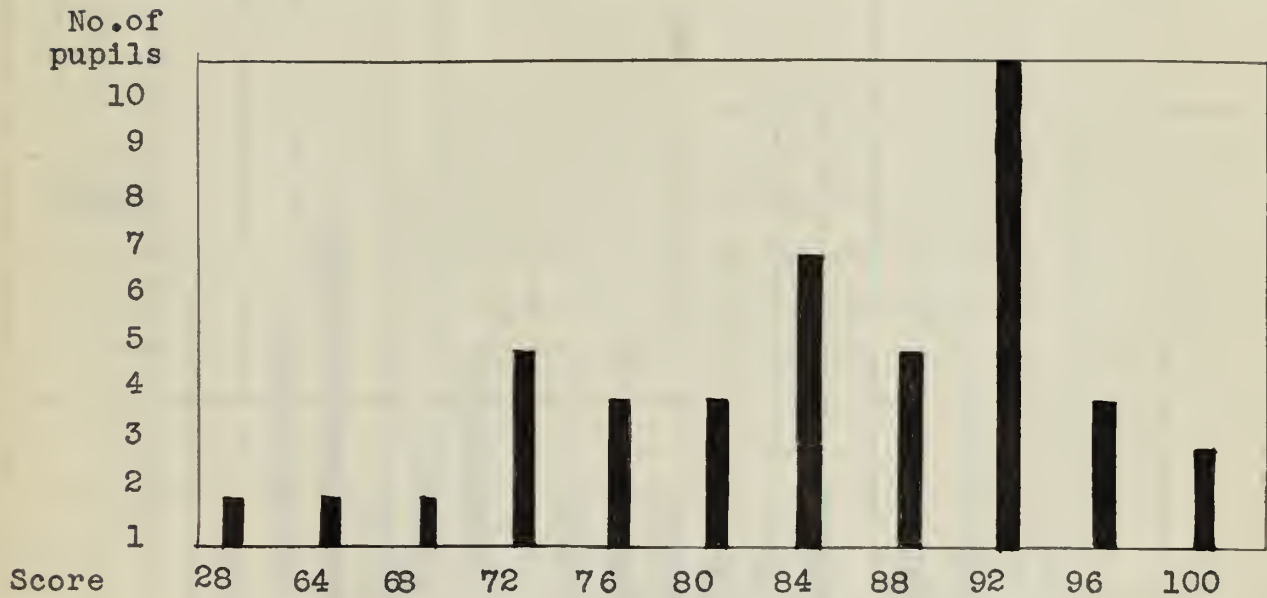


Figure 1 is read as follows:

2 pupils received a score of 100.



Figure 2

Showing graphically the distribution of time in the

5 P Multiplication Test made by 38 Fifth Grade pupils

Nov. 5, 1935

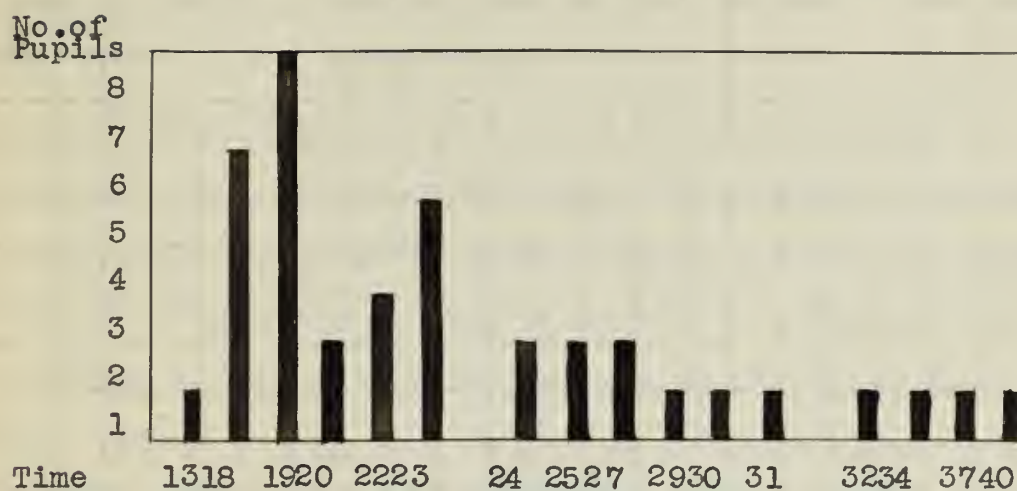


Figure 2 is read as follows:

1 pupil completed the test in 13 minutes.

All time included checking.





Eighteen types of errors were found in the 5 P Multiplication Test done by the 38 fifth grade pupils on Nov. 5, 1935. This was a gain of seven types of errors over the same test given to fifth grade pupils on Feb. 12, 1934.

"Carrying in multiplication" still seems to be the "bugbear." There were 57 errors of this type with a percentage of 36.37. "Omitted one partial product" came second with 16 errors and a percentage of 10.32. Third--"errors in adding the partial products"--14--9.03 %; fourth--"product in the wrong place"--10--6.45 %; fifth--"multiplied incorrectly--no carrying"--10--6.45 %; sixth--"added the partial products instead of multiplying them"--9--5.80 %; seventh--"multiplied instead of adding"--9--9.80%; eighth--"dollars and cents times 10" ( $\$5.90 \times 10$ )--5--3.22%; ninth--"zero times digit"--4--2.58%.

The tenth error was "omitted dollars sign and decimal point" with 4 errors--2.58 %; eleventh "multiplied by the wrong figure"--4--2.58 %; twelfth--"put in a decimal point"--3--1.93 %; thirteenth--"omitted a figure in the multiplicand"--3--1.93 %; fourteenth--"omitted dollars sign"--3--1.93 %; fifteenth--"digit times zero"--1--.64 %; sixteenth--"did not add the partial products"--1--.64 %; seventeenth--"put down the wrong figure"--1--.64 %; eighteenth--"subtracted instead of multiplying"--1--.64 %.

The total number of errors was 155.

The above data is shown in Table 1X which follows, and is shown graphically in Figure 3 following Table IX.

The first part of the paper is devoted to a discussion of the general principles of the theory of the structure of the atom. It is shown that the structure of the atom is determined by the laws of quantum mechanics, which are based on the principle of the uncertainty of the position and momentum of the particles.

The second part of the paper is devoted to a discussion of the structure of the nucleus. It is shown that the structure of the nucleus is determined by the laws of quantum mechanics, which are based on the principle of the uncertainty of the position and momentum of the particles.

The third part of the paper is devoted to a discussion of the structure of the molecule. It is shown that the structure of the molecule is determined by the laws of quantum mechanics, which are based on the principle of the uncertainty of the position and momentum of the particles.

The fourth part of the paper is devoted to a discussion of the structure of the crystal. It is shown that the structure of the crystal is determined by the laws of quantum mechanics, which are based on the principle of the uncertainty of the position and momentum of the particles.

The fifth part of the paper is devoted to a discussion of the structure of the liquid. It is shown that the structure of the liquid is determined by the laws of quantum mechanics, which are based on the principle of the uncertainty of the position and momentum of the particles.

The sixth part of the paper is devoted to a discussion of the structure of the gas. It is shown that the structure of the gas is determined by the laws of quantum mechanics, which are based on the principle of the uncertainty of the position and momentum of the particles.

The seventh part of the paper is devoted to a discussion of the structure of the plasma. It is shown that the structure of the plasma is determined by the laws of quantum mechanics, which are based on the principle of the uncertainty of the position and momentum of the particles.

The eighth part of the paper is devoted to a discussion of the structure of the solid. It is shown that the structure of the solid is determined by the laws of quantum mechanics, which are based on the principle of the uncertainty of the position and momentum of the particles.

The ninth part of the paper is devoted to a discussion of the structure of the liquid crystal. It is shown that the structure of the liquid crystal is determined by the laws of quantum mechanics, which are based on the principle of the uncertainty of the position and momentum of the particles.

Showing the Types, Number, and Percentage of Errors  
in the 5 P Multiplication Test, made by 38 Fifth Grade Pupils.

Nov. 5, 1935

1	Carrying in multiplication	57	36.77 %
2	Omitted one partial product	16	10.32 %
3	Errors in adding the partial products	14	9.03 %
4	Product in the wrong place	10	6.45 %
5	Multiplied incorrectly--no carrying	10	6.45 %
6	Added instead of multiplying	9	5.80 %
7	Multiplied instead of adding	9	5.80 %
8	Dollars and cents times 10 (\$5.90 X 10)	5	3.22 %
9	Zero times digit	4	2.58 %
10	Omitted dollars sign and decimal point	4	2.58 %
11	Multiplied by the wrong figure	4	2.59 %
12	Put in a decimal point	3	1.93 %
13	Omitted a figure in the multiplicand	3	1.93 %
14	Omitted the dollars sign	3	1.93 %
15	Digit times zero	1	.64 %
16	Did not add the products	1	.64 %
17	Put down the wrong figure	1	.64 %
18	Subtracted instead of multiplying	1	.64 %
Total number of errors		155	

A graph of this table follows.

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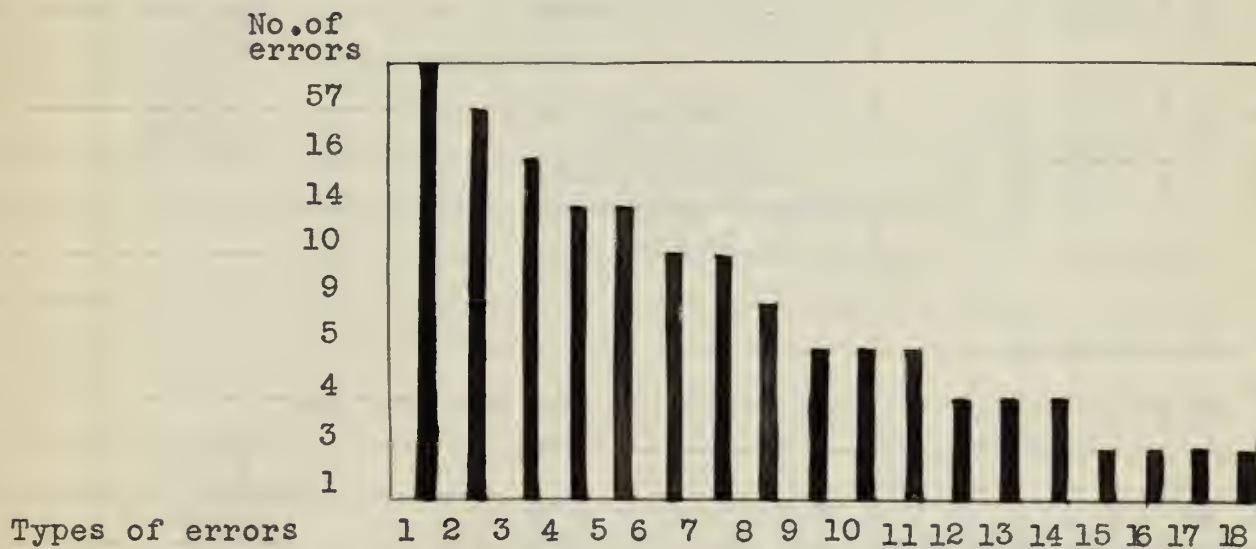


Figure 3

Showing Graphically the Types of Errors

in the 5 P Multiplication Test Made by 38 Fifth

Grade Pupils Nov. 5, 1935



The 18 types of errors appearing at the bottom of the graph are numbered to correspond with the numbers of the types of errors in Table IX.

Figure 3 is read as follows:

There were 57 errors of type No. 1 (See p. 25 )



In Table X which follows, the distribution of the types of errors has been tabulated according to examples. The twenty-five examples are numbered alphabetically from "a" to "y" inclusive. The first seven types of errors which numbered from 9 errors to 57 errors were quite generally distributed from example "h" to y inclusive.

Seven types of errors were found in example "w" as follows:

(w)	Carrying	12 errors
784	Omitted one product	4 errors
X 367	Error in adding	1 error
<u>5488</u>	Product in wrong	
4704	place	2 errors
2352	Multiplied	
<u>287728</u>	incorrectly	2 errors
	Added products	2 errors
	Multiplied instead	
	of adding	2 errors
		<hr/>
		25 errors

(t) \$ 680 120 <hr/> 13600 680 <hr/> \$81600	Example "w" had the greatest number of errors. There were 11 errors in example "t," 3 omitted the dollars sign and 3 put in a decimal point; 2 did not multiply by the 1, and one left out one of the figures in the multiplicand.
---	--

One partial product was omitted once in example "b," once in example "i," three times in example "n," twice in example "t" four times in example "v," four times in example "w," and once in example "y." There was one error in adding products in example "k," one in "m," one in "o," 2 in "q," two in "s," "t," "u," and "v" and one in "w." The other errors were scattering.

The first part of the document is a letter from the President of the United States to the Congress, dated January 1, 1861. The letter is signed by James Buchanan and is addressed to the Senate and House of Representatives. The letter discusses the state of the Union and the recent events leading to the secession of the Southern States.

Item	Quantity	Value
1. Cotton	100 bales	\$100,000
2. Sugar	50 tons	\$50,000
3. Rice	200 tons	\$200,000
4. Tobacco	100 tons	\$100,000
5. Wheat	500 tons	\$500,000
6. Corn	1,000 tons	\$1,000,000
7. Beans	100 tons	\$100,000
8. Peas	50 tons	\$50,000
9. Potatoes	1,000 tons	\$1,000,000
10. Apples	100 tons	\$100,000
11. Oranges	50 tons	\$50,000
12. Lemons	50 tons	\$50,000
13. Limes	50 tons	\$50,000
14. Bananas	100 tons	\$100,000
15. Pineapples	50 tons	\$50,000
16. Mangoes	50 tons	\$50,000
17. Guavas	50 tons	\$50,000
18. Avocados	50 tons	\$50,000
19. Papayas	50 tons	\$50,000
20. Jackfruits	50 tons	\$50,000

The second part of the document is a list of the goods and services provided to the President and his family during their stay in the White House. The list is dated January 1, 1861, and is signed by the President. The list includes items such as food, clothing, and other necessities. The total value of the goods and services is listed as \$1,000,000.

Table X

Showing the Distribution of the Types of Errors

by Examples in the 5 P Multiplication Test made by

38 Fifth Grade pupils Nov. 5, 1935

Ex.																			
a						1											1		2
b	1	1																	2
c						1													1
d				1															1
e					2			4								1			7
f				2															2
g								5											5
h	3																		3
i	1	1			1		2			1				1					7
j	2						1												3
k	5		1																6
l	3																		3
m	2		1		1														4
n	1	3							4										8
o	5		1													1			7
p	4																		4
q	5		2							1									8
r	4			3		1													8
s	2		2																4
t		2	2									3	1	3					11
u	5		2		1					1			1						10
v	1	4	2	2			2			1									12
w	12	4	1	2	2	2	2												25
x	1	1		2	2	2	1			1									10
y				1			1												2
Types of errors	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	155

Table X is read as follows:

Example a contains 2 errors--one in adding instead of multiplying (6) and one in subtracting instead of multiplying (18)





Since the I.Q.'s varied so much in this class it might be well to explain certain things about several of the pupils. Mary, with an I.Q. of 67, came to us from another district in the city. Her record of attendance was very poor. She had no mother. Her father was on the welfare. They made their home with her young married brother. When Mary came to us she could not add the simplest combinations. Before the 5 P multiplication was given she had received some individual help but not very much.

Her progress might have been discouraging to any but an understanding teacher. Her gains were slow but somewhat steady. On her first re-test, at the end of 45 twenty-minute periods, of teaching, most of which were given to individual instruction, she raised her score from 28--time 27 min.--to 68--time 19 min. On the second retest, following 6 twenty-minute periods of remedial work, her score dropped to 60--time 24 min. On her third re-test with 6 more periods of remedial work her score went up to 88--time 18 min. This was considered a triumph by her teacher, by the cadet teacher who also helped her, and by the writer. At the end of two more periods of twenty minutes each her score reached the 100 % mark--time 40 min. She had learned to check; this doubled her time.

Mary received 59 twenty-minute periods of remedial instruction--a total of 1180 minutes or 19 hours and 40 min-



utes. About half of the time was spent in individual instruction. The rest of the time she worked with Group 3.

Let us compare her record with that of Frances who had an I.Q. of 145 according to the National Intelligence Test. Frances was 9 years and 5 months of age when she took the test on Nov. 5, 1935. Her score was 92 and her time was 25 minutes. On the first re-test, after 10 twenty-minute periods of remedial work, she received a score of 88--time 15 minutes. She gained 10 minutes in time but lost 4 points on her score.

On the second re-test she received a score of 88, the same as on the first re-test, but shortened her time to 13 minutes, with an added 2 twenty-minute periods of remedial work. After 5 more twenty-minute periods of instruction she received a score of 96 on the third re-test--time 15 minutes. This was better than on the first test but her time was 2 minutes longer than on the second re-test.

On the fourth re-test, after having received a total of 50 twenty-minute periods of remedial work since Nov. 5, 1935, she received 92--time 12 minutes. She shortened her time but again lowered her score. On the successive re-tests she received scores of 88--time 15 minutes, 96--time 15 minutes, 96--time 15 minutes, and finally a score of 100--time 16 minutes, after having received 24 more twenty-minute periods of remedial work.

Frances 9 yr. 5 mo.--I.Q. 145--74 periods--1480 min.  
--24 hr. 40 min.





Mary 14 yr. 2 mo.--I.Q. 67--59 periods--1180 min.--  
19 hr. 40 min.

The two most extreme cases in the class have been cited here. It isn't quite possible to make a comparison as these girls have very different backgrounds and come from very different environments. Yet Mary's progress was one of the high lights of the year.

Another pupil in Group 3 deserves some mention. Geneva was 12 years and 1 month of age when she took the test on Nov. 5. Her score was 72 and her time 27 minutes. Her I.Q., based on the National Intelligence Test was 86. The Binet test given last year gave her an I.Q. of 79. She reached her record of 100 % accuracy after having received 45 twenty-minute periods of remedial work or a total of 900 minutes equaling 15 hours.

Geneva is the type of pupil, who, if given time enough, can learn on a drill basis. Her record was very good in comparison with other members of the class. There were 6 pupils with higher I.Q.'s who received fewer periods of remedial work than she; there were eight others who received the same number of periods of remedial work, which was 45; and there were 21 who received more periods of remedial work than she.

Josephine and Catherine are twins. Josephine has an I.Q. of 87 and Catherine an I.Q. of 78 according to the National Intelligence Test. Both of these girls were retarded in the

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fourth grade before they came to our building. They are of Italian parentage and have had the handicap of language to overcome. They were 11 years and 2 months of age when the test was given on Nov. 5, 1935. Josephine received a score of 100--time 20 min. She was one of two pupils in the class to receive a score of 100, on the first test. Catherine received a score of 92--time 19 minutes. After 50 twenty-minute periods of remedial work Catherine received her perfect score. She was re-tested 4 times.

Josephine--age 11-2 I.Q.87 100--23 min.

Catherine--age 11-2 I.Q.78 92--19 min. 50 per. 1000 min.  
16 hr. 40 min.

For convenience in handling the pupils either for group work or individual instruction the thirty-six pupils not receiving perfect scores on the initial test were divided into three groups. The first group consisted of the pupils receiving scores of 96 and 92. The second group was made up of pupils receiving scores of 88, 84, and 80. The third group consisted of the pupils receiving scores of 76, 72, 68, 64, and 28.

On the first re-test in Group 1 one pupil received a perfect score after 10 twenty-minute periods of remedial work; in Group 2 no one received a perfect score; in Group 3 three pupils received perfect scores after 45 twenty-minute periods of instruction. On the second re-test in Group 1 three pupils received perfect scores after 2 more twenty-

18  
The first thing I noticed when I stepped out of the car was the smell of the sea. It was a salty, fresh scent that I had never experienced before. The sun was shining brightly, and the waves were crashing against the shore. I felt a sense of freedom and adventure as I walked along the beach. The sand was soft and warm under my feet, and the breeze was a pleasant surprise. I had heard that the weather was perfect, and now I knew why. It was indeed a beautiful day, and I was lucky to be here.

I had heard that the weather was perfect, and now I knew why. It was indeed a beautiful day, and I was lucky to be here. The beach was crowded, but not too crowded. There were people of all ages and backgrounds, all enjoying the sun and the sand. I saw a group of children playing in the water, their laughter echoing across the beach. I also saw a couple walking hand in hand, their love evident in their smiles. The beach was a place of joy and relaxation, and I was glad to be a part of it. I had heard that the weather was perfect, and now I knew why. It was indeed a beautiful day, and I was lucky to be here.

I had heard that the weather was perfect, and now I knew why. It was indeed a beautiful day, and I was lucky to be here. The beach was crowded, but not too crowded. There were people of all ages and backgrounds, all enjoying the sun and the sand. I saw a group of children playing in the water, their laughter echoing across the beach. I also saw a couple walking hand in hand, their love evident in their smiles. The beach was a place of joy and relaxation, and I was glad to be a part of it. I had heard that the weather was perfect, and now I knew why. It was indeed a beautiful day, and I was lucky to be here.



minute periods of help. In Group 2 five pupils received perfect scores after 45 twenty-minute periods of remedial work; and in Group 3 one pupil received a perfect score after 51 twenty-minute periods of instruction.

On the third re-test in Group 1 one pupil received a perfect score; in Group 2 one pupil received a perfect score after 60 twenty-minute periods of help; and in Group 3 no one received a perfect score. On the fourth re-test in Group 1 one pupil received a perfect score; in Group 2 no one received a perfect score; and in Group 3 three pupils received perfect scores.

On the fifth re-test in Group 1 three received perfect scores after 51 twenty-minute periods of instruction and one pupil received a perfect score after 50 twenty-minute periods of help; in Group 2 one received a perfect score after receiving help for 68 twenty-minute periods; and in Group 3 no one received a perfect score. On the sixth re-test in Group 1 two pupils received perfect scores with a total of 66 twenty-minute periods; in Group 2 two pupils received perfect scores with a total of 72 twenty-minute periods; and in Group 3 the last three pupils in the group received perfect scores with a total of 59 twenty-minute periods of remedial work.

On the seventh re-test no one in either the first or second group received perfect scores. On the eighth re-test one pupil in Group 1 received a perfect score after a total





of 74 twenty-minute periods of remedial work; and in Group 2 two pupils received perfect scores, one after having received 74 twenty-minute periods of help and the other after having received 44 twenty-minute of remedial work.

1742 twenty-minute periods equalling 580 hours and 40 minutes were used in this remedial work with 36 pupils. The "tables" habit is a very tenacious one. This probably accounts for the length of time it took to attain perfect scores. Most of these fifth grade pupils were taught the "tables" through the 6's in the third grade and the remaining "tables" through the 12's in the fourth grade.

It was a joy to work with these children. As each pupil received a perfect score he or she dropped out of the drill group. They were commended for their achievement and those who remained in the group were encouraged to try to do better the next time. At no time was coercion used. This was done as extra work. Many times children came in early for extra help and many remained voluntarily for more work after the others had gone home. A definite record was kept of all instruction periods, whether individual or group work, and these results are assembled in Table XI which follows:



Table XI

Showing the scores made and the time used in taking the first test and the total time allotted to Remedial work with 38 fifth grade pupils between Nov. 5, 1935 and June 23, 1936.

Nov. 5, 1935			First Re-test Dec. 17, 1935				Second Re-test Dec.20,1935		Third Re-test Jan.10,1936			Fourth Re-test Jan.17,1936			Fifth Re-test Apr.1,1936			Sixth Re-test May 19,1936			Seventh Re-test June 3, 1936			Eighth Re-test June 11, 1936			June 19, 1936			June 22, 1936			June 23, 1936												
Age	I.Q. N.I.T.	Score	Time	Score	Time	No. of 20 min. periods	Score	Time	No. of 20 min. periods	Score	Time	No. of 20 min. periods	Score	Time	No. of 20 min. periods	Score	Time	No. of 20 min. periods	Score	Time	No. of 20 min. periods																								
1 Beatrice	10- 0	133	100	23																																									
2 Josephine	11- 2	87	100	20																																									
3 Dorothy B.	11- 8	87	96	22	96	23	10			100	28	2																																	
4 Esther	12- 5	89	96	40	88	22	10			92	29	2	88	20	5	88	19	4	100	25	30																								
5 Alfred	10-10	105	96	30	96	22	10			92	19	2	96	20	5	100	19	4																											
6 Rose T.	10- 2	121	92	19	92	23	10			100	27	2																																	
7 Anna C.	10- 5	111	92	22	88	15	10			92	14	2	96	16	5	92	13	4	100	11	30																								
8 Lenora	9- 9	121	92	20	92	19	10			100	18	2																																	
9 Ruth S.	10- 3	115	92	18	100	19	10												100	22	15																								
10 Rita	11- 3	101	92	23	88	21	10			92	16	2	92	19	5	92	17	4	96	16	30																								
11 Erio	10- 0	132	92	23	84	22	10			80	17	2	100	16	5				88	15	15	96	15	6	100	16	3																		
12 Frances	9- 5	145	92	25	88	15	10			88	13	2	96	15	5	Ab.		3	92	12	30	100	13	15																					
13 Salvatore	10- 3	105	92	23	68	14	10			84	14	2	88	15	5	92	10	4	94	15	30																								
14 Hazel	10- 9	106	92	18	88	15	10			92	13	2	96	16	5	96	11	4	100	17	30																								
15 Catherine	11- 2	79	92	19	92	17	10			96	14	2	96	16	5	Ab.		3	100	16	30																								
																									Third Re-test			Fourth Re-test			Fifth Re-test			Sixth Re-test			Seventh Re-test			Eighth Re-test					
																									Begun Dec. 1, 1935			First Re-test			Second Re-test														
16 Ethel	10- 4	110	88	25						88	14	15	100	12	30																														
17 Elinor	9- 0	107	88	19						96	14	15	100	16	30																														
18 Joseph	10- 5	115	88	27						88	19	15	100	16	30																														
19 Fred	10- 3	116	88	27						80	13	15	Absent			88	7	15	96	11	6	92	14	3	92	16	3																		
20 Barbara	9-11	114	84	18						96	25	15	100	10	30																														
21 Mary G.	10- 5	113	84	19						96	12	15	92	12	30				80	11	15	84	14	6	88	12	3	100	22	3															
22 John F.	11- 3	105	84	22						64	14	15	96	13	30				76	16	15	84	15	6	92	12	3	100	10	3															
23 Jack	11- 1	104	84	29						Ab.		10	Ab.		20	100	21	15																											
24 Rose C.	11- 3	100	84	24						88	21	15	96	27	30				100	22	15																								
25 Lillian	10- 9	133	80	13						84	23	15	100	17	30																														
26 Everett	10- 9	133	80	24						68	18	15	96	13	30				80	14	15	88	13	6	92	12	3	92	10	3															
27 Gerald	12- 7	83	80	13						92	9	15	96	10	30				96	7	15	96	10	6	100	15	3																		
28 Robert S.	10-11	93	80	19						96	13	15	Ab.		25	100	16	15																											









Table XII

Showing a Summary of the scores made and the time used in taking the test first test and the total time allotted to remedial work with 38 fifth grade pupils between Nov. 5, 1935 and June 23, 1936.

	Age	I.Q.	N.I.T.	Score	Time	No. of 20 min. periods	Total No. Minutes	Hours ) and ) ) Minutes	
1	Beatrice	10- 0	: 133	: 100	: 23	:	:	:	:
2	Josephine	11- 2	: 87	: 100	: 20	:	:	:	:
3	Dorothy B.	11- 8	: 87	: 96	: 22	: 10	: 200	: 3	: 20
4	Esther	12- 5	: 89	: 96	: 40	: 51	: 1020	: 17	:
5	Alfred	10-10	: 105	: 96	: 30	: 21	: 420	: 7	:
6	Rose	10- 2	: 121	: 96	: 19	: 12	: 240	: 4	:
7	Anna C.	10- 5	: 111	: 92	: 22	: 51	: 1020	: 17	:
8	Lenora	9- 9	: 121	: 92	: 20	: 12	: 240	: 4	:
9	Ruth S.	10- 3	: 115	: 92	: 18	: 10	: 200	: 3	: 20
10	Rita	11- 3	: 101	: 92	: 23	: 67	: 1340	: 22	: 20
11	Eric	10- 0	: 132	: 92	: 23	: 17	: 340	: 5	: 40
12	Frances	9- 5	: 145	: 92	: 25	: 74	: 1480	: 24	: 40
13	Salvatore	10- 3	: 105	: 92	: 23	: 66	: 1320	: 22	:
14	Hazel	10- 9	: 106	: 92	: 18	: 51	: 1020	: 17	:
15	Catherine	11- 2	: 78	: 92	: 19	: 50	: 1000	: 16	:
16	Ethel	10- 4	: 110	: 88	: 25	: 45	: 900	: 15	:
17	Elinor	9- 0	: 107	: 88	: 19	: 45	: 900	: 15	:
18	Joseph	10- 5	: 115	: 88	: 27	: 45	: 900	: 15	:
19	Fred	10- 3	: 116	: 88	: 18	: 44	: 880	: 14	: 40
20	Barbara	9-11	: 114	: 84	: 18	: 45	: 900	: 15	:
21	Mary G.	10- 5	: 113	: 84	: 19	: 72	: 1440	: 24	:
22	John F.	11- 3	: 105	: 84	: 22	: 72	: 1440	: 24	:
23	Jack	11- 1	: 104	: 84	: 29	: 45	: 900	: 15	:
24	Rose C.	11- 3	: 100	: 84	: 24	: 60	: 1200	: 20	:
25	Lillian	10- 7	: 115	: 84	: 31	: 45	: 900	: 15	:
26	Everett	10- 9	: 133	: 80	: 24	: 74	: 1480	: 24	: 40
27	Gerald	12- 7	: 83	: 80	: 13	: 69	: 1380	: 23	:
28	Robert S.	10-11	: 93	: 80	: 19	: 55	: 1100	: 18	:

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53	54	55	56
57	58	59	60
61	62	63	64
65	66	67	68
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73	74	75	76
77	78	79	80
81	82	83	84
85	86	87	88
89	90	91	92
93	94	95	96
97	98	99	100

Table XII Continued

		Age	I.Q.	N.I.T.	Score	Time	No. of 20 min. periods	Total No. minutes	Hours ) and ) Minutes	
29	Beatrice H.	12- 8	: 99	: 76	: 19	: 45	: 900	: 15	:	:
30	John D.	10- 0	: 113	: 76	: 18	: 51	: 1020	: 17	:	:
31	Douglas	10- 8	: 118	: 76	: 18	: 59	: 1180	: 19	: 40	:
32	Peggy	9- 3	: 131	: 72	: 34	: 57	: 1140	: 19	:	:
33	Alice	11- 8	: 100	: 72	: 32	: 57	: 1140	: 19	:	:
34	Dorothy M.	11- 2	: 107	: 72	: 23	: 45	: 900	: 15	:	:
35	Geneva	12- 1	: 86	: 72	: 27	: 45	: 900	: 15	: 40	:
36	Florence	11- 5	: 108	: 68	: 19	: 59	: 1180	: 19	: 40	:
37	Thomas	10- 5	: 122	: 64	: 18	: 57	: 1140	: 19	:	:
38	Mary R.	14- 2	: 67	: 28	: 37	: 59	: 1180	: 19	: 40	:
Average							48 :	940 :	15 :	40 :

An average of 48 twenty-minute periods of remedial work was given to each fifth grade pupil or 15 hr. 40 min.



STATE OF NEW YORK

IN SENATE									
JANUARY 18, 1890									
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

REPORT OF THE COMMISSIONERS OF THE LAND OFFICE  
IN RESPONSE TO A RESOLUTION PASSED BY THE SENATE  
JANUARY 18, 1890

One of the first skills that the fifth grade has to develop is the skill of checking. This is taught in our own fourth grades, but a full class of fifth grade pupils comes to us in September from another district. These pupils may or may not have been required to check in the fourth grade. All of the time recorded includes checking as stated earlier in the study. All of these pupils were taught the multiplication combinations by the "tables" method. So the writer feels, very conclusively, that when the time was long, counting or "saying the tables" resulted. Increase in time means a very definite form of remedial instruction to change this particular habit of work.

The remedial work was carried on by a very competent room teacher for the most part. Some of the work with Group 3 was carried on by a cadet teacher under the direction of the regular room teacher. All of the testing was done by the writer and occasionally some of the remedial work also.

The first step in the remedial work was to find out in which combinations each pupil was weak. Drill was given on these combinations after they were first taught by groups according to the Wilson Drill service. Then the steps were taken in the order of difficulty. Each pupil moved to the next step or to the next group just as fast as the work was accomplished with perfect mastery.



The set up of the Wilson Drill Service is explained in the foreword to the pupil:

"Multiplication has been a favorite process. The number of facts needed is small, a total of only one hundred. In business it is the process most used. You will want therefore to master multiplication perfectly. Results less than 100% are not satisfactory.

The general plan in this book is to divide the facts into small groups. There are ten groups. You master the facts of the group and then you learn to use the facts in bigger examples. As a result, when you have worked all the examples in this drill book you should know multiplication perfectly. It is not a difficult process. It uses addition and so gives you a good review of previous work in addition.

You will want to learn each fact so well that you know it standing alone whenever you meet it. There will be opportunities for checking yourself, thus giving you a chance to see how well you are getting along. You will enjoy the game of multiplication. Every child can win." (48)

In the Wilson Drill Service the 100 multiplication combinations are divided into ten groups. Each group is made up of ten facts which are really five facts and their reversals. Each group is divided into ten steps proceeding according to difficulty. The first steps contain the easier processes in multiplication and proceed through the ten steps with the processes becoming more difficult.

One group--Group 1--with the ten steps of difficulty of the Wilson Drill Service follows in Table XIII.





Table XIII

From the Wilson Drill Service

41

## Group I--MULTIPLICATION (48)

Steps 1 and 2. Primary Facts.

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
7	3	7	1	5	0	2	5	2	2
0	3	5	2	2	7	2	7	1	5
0	9	35	2	10	0	4	35	2	10

All of the work in Group 1 will involve only the combinations used in Group 1

Multiply:

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	
1	7	2	7	5	0	2	3	(1)
2	5	5	0	2	7	5	3	
5	0	3	2	5	2	7	7	(2)
2	7	3	2	7	1	0	0	
7	7	2	3	1	0	2	5	(3)
0	5	1	3	2	7	2	7	
2	1	5	5	2	7	3	2	(4)
2	2	7	2	1	5	3	5	

## Step 3--One-Place Multiplier, No Carrying

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
1 1	1 1 1	2 2	\$2.2 2	1 2	2 1	1 2 2	2 1 2
2	2	2	2	2	2	2	2
(i)	(j)	(k)	(l)	(m)	(n)	(o)	(p)
2 2 1	\$1.1 2	1 2 1	2 1 1	3 3	\$3.3 3	2 2	2 2 2
2	2	2	2	2	2	2	2

## Step 4. One-Place Multiplier, Carrying Requiring Addition in same Decade

(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
5 5	\$5.5 5	7 7	7 7 7	2 2	\$2.2 2	7 2	\$ .2 7
7	7	5	5	5	5	5	5
(i)	(j)	(k)	(l)	(m)	(n)	(o)	(p)
7 7 2	7 2 7	7 7 2	7 2 2	2 7 2	2 2 7	5 5	\$5.5 5
5	5	5	5	5	5	2	2

Table 1

Table 1 shows the results of the first two experiments. The first experiment was a 2x2x2 factorial design with factors of stimulus type (visual vs. auditory), stimulus duration (100 ms vs. 200 ms), and stimulus intensity (10 dB vs. 20 dB). The second experiment was a 2x2x2 factorial design with factors of stimulus type (visual vs. auditory), stimulus duration (100 ms vs. 200 ms), and stimulus intensity (10 dB vs. 20 dB).

Stimulus Type	Stimulus Duration	Stimulus Intensity	Mean Response Time (ms)	Standard Error (ms)
Visual	100 ms	10 dB	150	10
Visual	100 ms	20 dB	160	10
Visual	200 ms	10 dB	170	10
Visual	200 ms	20 dB	180	10
Auditory	100 ms	10 dB	140	10
Auditory	100 ms	20 dB	150	10
Auditory	200 ms	10 dB	160	10
Auditory	200 ms	20 dB	170	10

Table 1 shows the results of the first two experiments. The first experiment was a 2x2x2 factorial design with factors of stimulus type (visual vs. auditory), stimulus duration (100 ms vs. 200 ms), and stimulus intensity (10 dB vs. 20 dB). The second experiment was a 2x2x2 factorial design with factors of stimulus type (visual vs. auditory), stimulus duration (100 ms vs. 200 ms), and stimulus intensity (10 dB vs. 20 dB).

Stimulus Type	Stimulus Duration	Stimulus Intensity	Mean Response Time (ms)	Standard Error (ms)
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Visual	100 ms	20 dB	160	10
Visual	200 ms	10 dB	170	10
Visual	200 ms	20 dB	180	10
Auditory	100 ms	10 dB	140	10
Auditory	100 ms	20 dB	150	10
Auditory	200 ms	10 dB	160	10
Auditory	200 ms	20 dB	170	10

Table 1 shows the results of the first two experiments. The first experiment was a 2x2x2 factorial design with factors of stimulus type (visual vs. auditory), stimulus duration (100 ms vs. 200 ms), and stimulus intensity (10 dB vs. 20 dB). The second experiment was a 2x2x2 factorial design with factors of stimulus type (visual vs. auditory), stimulus duration (100 ms vs. 200 ms), and stimulus intensity (10 dB vs. 20 dB).

Stimulus Type	Stimulus Duration	Stimulus Intensity	Mean Response Time (ms)	Standard Error (ms)
Visual	100 ms	10 dB	150	10
Visual	100 ms	20 dB	160	10
Visual	200 ms	10 dB	170	10
Visual	200 ms	20 dB	180	10
Auditory	100 ms	10 dB	140	10
Auditory	100 ms	20 dB	150	10
Auditory	200 ms	10 dB	160	10
Auditory	200 ms	20 dB	170	10

Table 1 shows the results of the first two experiments. The first experiment was a 2x2x2 factorial design with factors of stimulus type (visual vs. auditory), stimulus duration (100 ms vs. 200 ms), and stimulus intensity (10 dB vs. 20 dB). The second experiment was a 2x2x2 factorial design with factors of stimulus type (visual vs. auditory), stimulus duration (100 ms vs. 200 ms), and stimulus intensity (10 dB vs. 20 dB).

Stimulus Type	Stimulus Duration	Stimulus Intensity	Mean Response Time (ms)	Standard Error (ms)
Visual	100 ms	10 dB	150	10
Visual	100 ms	20 dB	160	10
Visual	200 ms	10 dB	170	10
Visual	200 ms	20 dB	180	10
Auditory	100 ms	10 dB	140	10
Auditory	100 ms	20 dB	150	10
Auditory	200 ms	10 dB	160	10
Auditory	200 ms	20 dB	170	10

Step 5 One-Place Multiplier, Carrying Requiring Addition into Higher Decade

Because the multipliers in Group 1 are all small there are no examples under Step 5. You will have no practice on Step 5, therefore, until you reach Step 5 of Group 11.

Step 6. One-Place Multiplier, Zero in Multiplicand, with and Carrying to the Zero

(a)	(b)	(c)	(d)
\$ . 5 0	5 0 5	5 0 5 5	\$5 5 . 0 5
7	7	7	7

Step 7. Two- or Three-Place Multiplier, No Carrying

(a)	(b)	(c)	(d)	(e)	(f)	(g)
1 1	1 1 1	2 2	\$2.2 2	1 2	2 1	1 2 2
2 2	2 2 2	2 2	2 2 2	2 2	2 2	2 2 2

(h)	(i)	(j)	(k)	(l)	(m)	(n)
2 1 2	\$2.2 1	1 1 2	1 2 1	2 1 1	\$.3 3	3 3 3
2 2	2 2	2 2	2 2	2 2 2	3 3	3 3 3

(o)	(p)	(q)	(r)	(s)	(t)	(u)
2 2	2 2 2	2 2	2 2	2 2 2	2 2 2	1 1 2
1 1	1 1 1	1 2	2 1	1 2 2	1 2 1	2 2

Step 8. Two- or Three-Place Multiplier, with Carrying

(a)	(b)	(c)	(d)	(e)	(f)	(g)
5 5	5 5 5	\$.7 7	7 7 7	2 2	\$2.2 2	2 7
7 7	7 7 7	5 5	5 5 5	5 5	5 5 5	5 5

(h)	(i)	(j)	(k)	(l)	(m)	(n)
7 2	2 2 7	2 7 2	7 2 2	2 7 7	7 2 7	7 7 2
5 5	5 5	5 5	5 5	5 5	5 5 5	5 5

(o)	(p)	(q)	(r)	(s)	(t)	(u)
5 5	5 5 5	2 5	\$2.5 5	5 2 5	5 5 2	2 5 2
2 2	2 2 2	2 2	2 2	2 2	2 2 2	2 2

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## Step 9. Single Zero in the Multiplier

(a)	(b)	(c)	(d)	(e)
$\begin{array}{r} 77 \\ \underline{50} \end{array}$	$\begin{array}{r} \$7.77 \\ \underline{50} \end{array}$	$\begin{array}{r} 77 \\ \underline{505} \end{array}$	$\begin{array}{r} 777 \\ \underline{5055} \end{array}$	$\begin{array}{r} \$7.77 \\ \underline{5505} \end{array}$

## Step 10. Double Zeros in the Multiplicand and Multiplier

(a)	(b)	(c)	(d)
$\begin{array}{r} 500 \\ \underline{77} \end{array}$	$\begin{array}{r} 5005 \\ \underline{777} \end{array}$	$\begin{array}{r} 777 \\ \underline{500} \end{array}$	$\begin{array}{r} 777 \\ \underline{5005} \end{array}$





Drill was given very specifically on the multiplication combinations that were weak. Almost every error can be traced to its relative position in the Drill Service by finding the group and the step in that particular group that carries the difficulty encountered by the pupil. Then drill is given in that step of that particular group.

A great deal of flash card drill is used for automatic response. Each card contains one of the one hundred multiplication combinations. On the reverse side of the card is the same combination with the answer. If a pupil hesitates on a combination he is given the card where he may study the particular combination on which he failed.

Games with flash cards may be introduced. The pupil wins the game who has no cards in his hands at the end of the drill. Some times all in the group are winners and that causes fun. At odd moments during the day, or in out of school hours the children like to help each other with the flash card drill on the combinations. Several groups of two, one flashing the cards and one answering, may have a race. The group finishing first becomes the winner.

Appendix B contains the 100 multiplication combinations used in flash card drill.

Another type of flash card drill is the kind that contains the multiplication combination and the carrying figure such as  $6 \times 5 + 4$ . This type of drill is in Appendix C.



## Chapter VI

Specific Remedial Work in Grade Six  
and the Results

The story of specific remedial work in the sixth grade will be of interest. The work was carried on in a similar manner to the work done in the fifth grade. The 5 P Multiplication Test was given in the sixth grade on April 3, 1936. There were 38 pupils in this class also. Their ages ranged from 10-7 to 12-11. I.Q.'s ranged from 92 (low) to 138 (high).

The lowest score was 80 and the highest 100. Eleven pupils received perfect scores. The shortest time was ten minutes and the longest time was twenty-seven minutes. The data on scores and time follows on page 46 Table XIV. The same data is shown graphically on pages 47 and 48 Figures 4 and 5.

# THE HISTORY OF THE UNITED STATES

OF AMERICA

FROM THE FIRST SETTLEMENTS TO THE PRESENT TIME

BY J. B. HARRIS, ESQ., OF NEW-YORK

IN THREE VOLUMES. VOL. I. FROM THE FIRST SETTLEMENTS TO THE END OF THE SEVENTEENTH CENTURY. NEW-YORK: PUBLISHED BY J. B. HARRIS, 1795.

THE HISTORY OF THE UNITED STATES OF AMERICA, FROM THE FIRST SETTLEMENTS TO THE PRESENT TIME, BY J. B. HARRIS, ESQ., OF NEW-YORK. IN THREE VOLUMES. VOL. I. FROM THE FIRST SETTLEMENTS TO THE END OF THE SEVENTEENTH CENTURY. NEW-YORK: PUBLISHED BY J. B. HARRIS, 1795.



Table XIV

Showing the Scores made on the 5 P Multiplication  
Test by 38 Sixth Grade Pupils, April 3, 1936

<u>No. of pupils</u>	<u>Score</u>	
11-----	100-----	Q <sub>3</sub> -----100
9-----	96-----	Median--- 96
10-----	92-----	Q <sub>1</sub> ----- 92
4-----	88-----	
2-----	84-----	
2-----	80-----	
<u>38</u>		
Standard Deviation 6.05		

The Time taken by 38 pupils in the Sixth Grade on the 5 P  
Multiplication Test is indicated below:

<u>No. of pupils</u>	<u>Time</u>	
3-----	10-----	
6-----	11-----	
2-----	12-----	Q <sub>3</sub> -----12
10-----	13-----	Median--13
3-----	14-----	
1-----	15-----	
1-----	16-----	
3-----	17-----	
2-----	18-----	Q <sub>1</sub> -----18
2-----	19-----	
1-----	20-----	
2-----	23-----	
2-----	27-----	
<u>38</u>		



Figure 4

Showing graphically the distribution of scores  
on the 5 P Multiplication Test made by 38 Sixth Grade

Pupils April 3, 1936

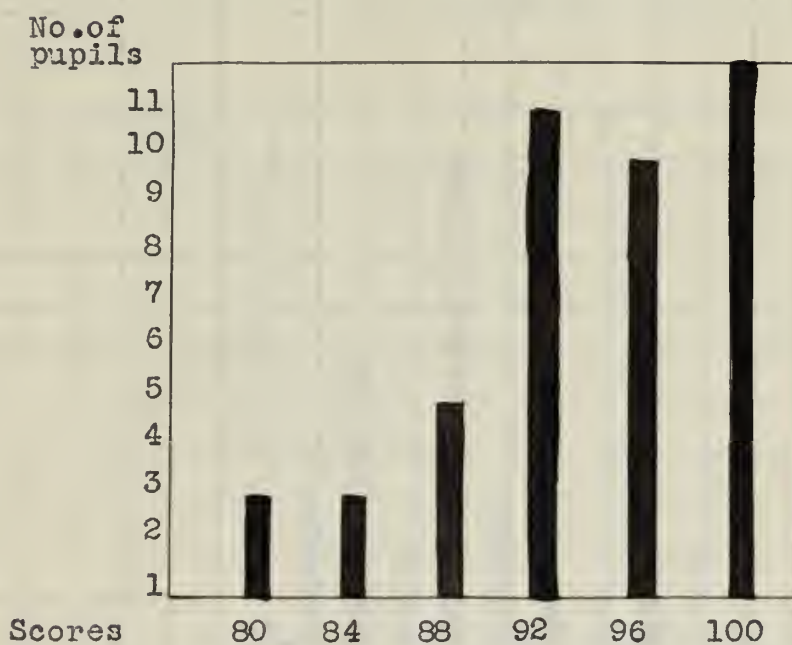


Figure 4 is read as follows:

11 pupils received a score of 100

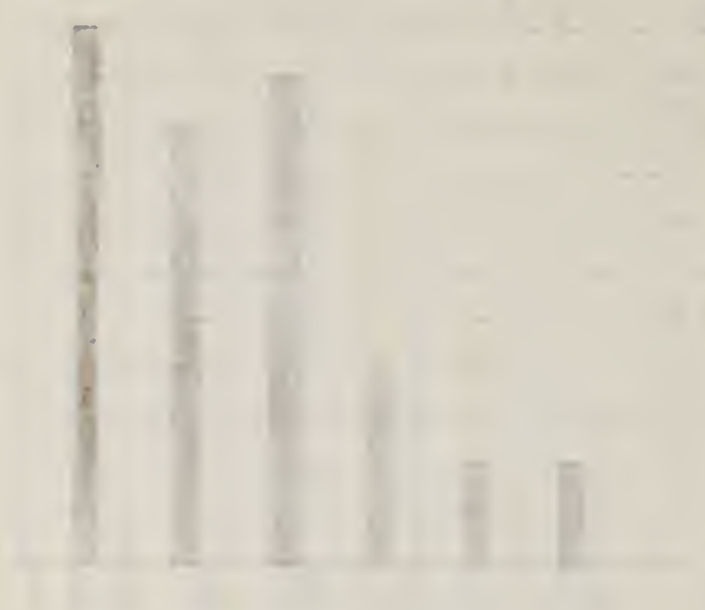


Figure 5

Showing graphically the distribution of time in the  
5 P Multiplication Test made by 38 Sixth Grade Pupils

April 3, 1936

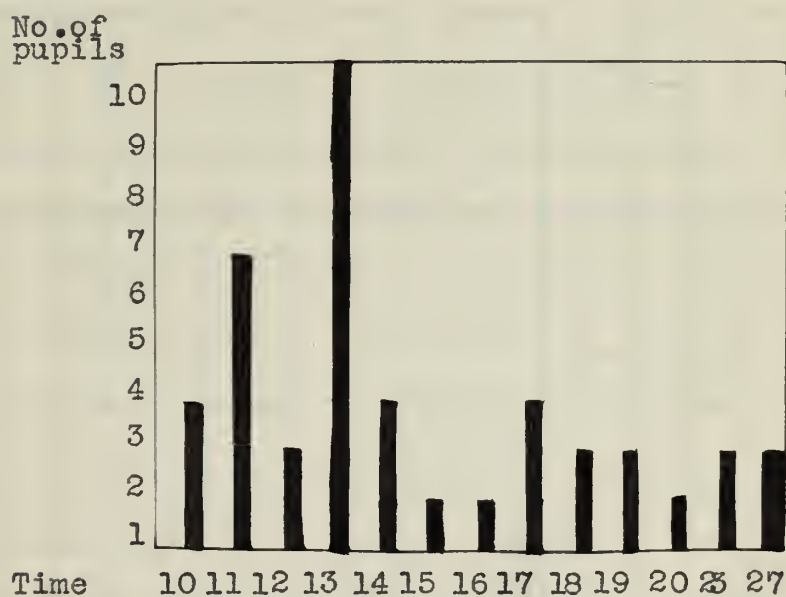
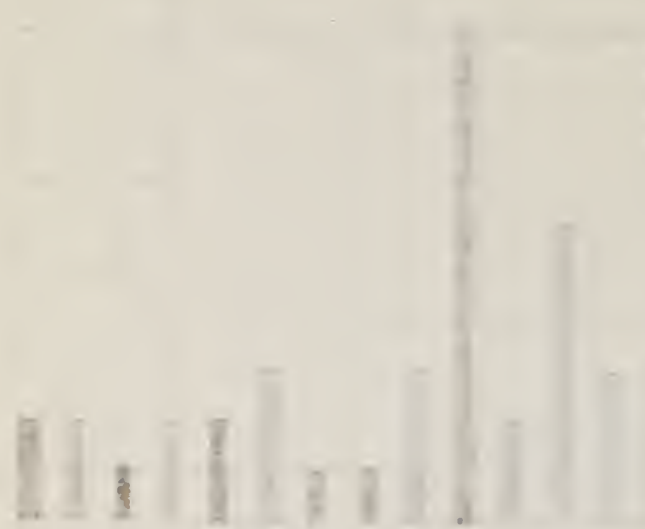


Figure 5 is read as follows:

3 pupils completed the test in 10 minutes.

All time included checking.





Thirteen types of errors were found in the 5 P Multiplication test taken by the 38 sixth grade pupils on April 3, 1936. This was a gain on 5 over the number of types of errors when the test was given to 38 fifth grade pupils Nov. 5, 1935.

"Carrying in Multiplication" remains at the head of the list of errors with 13--22.01 %. Second on the list is "Product in the wrong place"--10 errors--17.01 %; third "added instead of multiplying"--6 errors--10.16 %; fourth "dollars and cents times 10" ( \$5.90 X 10)--6 errors--10.16 %; fifth "multiplied wrong--no carrying"--5 errors--8.47 %; sixth "error in adding the partial products"--4 errors--6.61 %; seventh "digit times zero"--4 errors--6.61 %; eighth "multiplied instead of adding"--3 errors--5.08 %; ninth "omitted one product"--2 errors--3.38 %; tenth "omitted decimal point"--2 errors--3.38 %; eleventh "omitted a figure in the multiplicand"--2 errors--3.38 %; twelfth "omitted an example"--1 error--1.69 %; thirteenth "multiplied by the wrong figure"--1 error--1.69%.

The total number of errors was 59.

The above data is shown in Table XV which follows, and is shown graphically in Figure 6 which follows Table XV.

1. The first part of the paper discusses the importance of the study and the objectives of the research. It also mentions the scope of the study and the limitations. The second part of the paper discusses the methodology used in the study. It mentions the data sources and the statistical methods used. The third part of the paper discusses the results of the study. It mentions the findings and the conclusions. The fourth part of the paper discusses the implications of the study. It mentions the practical applications and the future research. The fifth part of the paper discusses the conclusion. It mentions the overall findings and the recommendations. The sixth part of the paper discusses the references. It mentions the sources used in the study. The seventh part of the paper discusses the appendix. It mentions the additional information provided. The eighth part of the paper discusses the index. It mentions the location of the topics in the paper. The ninth part of the paper discusses the glossary. It mentions the definitions of the terms used. The tenth part of the paper discusses the bibliography. It mentions the sources used in the study.

Table XV

Showing the types, number, and percentage of errors in the 5 P Multiplication Test made by 38 Sixth Grade pupils, April 3, 1936.

The types, number, and percentage of errors has decreased in this sixth grade. 13 types of errors were found.

1	Carrying in multiplication	13	22.01 %
2	Product in the wrong place	10	17.01 %
3	Added instead of multiplying	6	10.16 %
4	Dollars and cents times 10	6	10.16 %
5	Multiplied wrong-no carrying	5	8.47 %
6	Error in adding	4	6.61 %
7	Digit times zero	4	6.61 %
8	Multiplied instead of adding	3	5.08 %
9	Omitted one product	2	3.38 %
10	Omitted decimal point	2	3.38 %
11	Omitted a figure in the multiplicand	2	3.38 %
12	Omitted an example	1	1.69 %
13	Multiplied by the wrong figure	1	1.69 %
Total number of errors		<hr/> 59	

# THE JOURNAL OF THE AMERICAN MEDICAL ASSOCIATION PUBLISHED WEEKLY CHICAGO, ILL., U.S.A.

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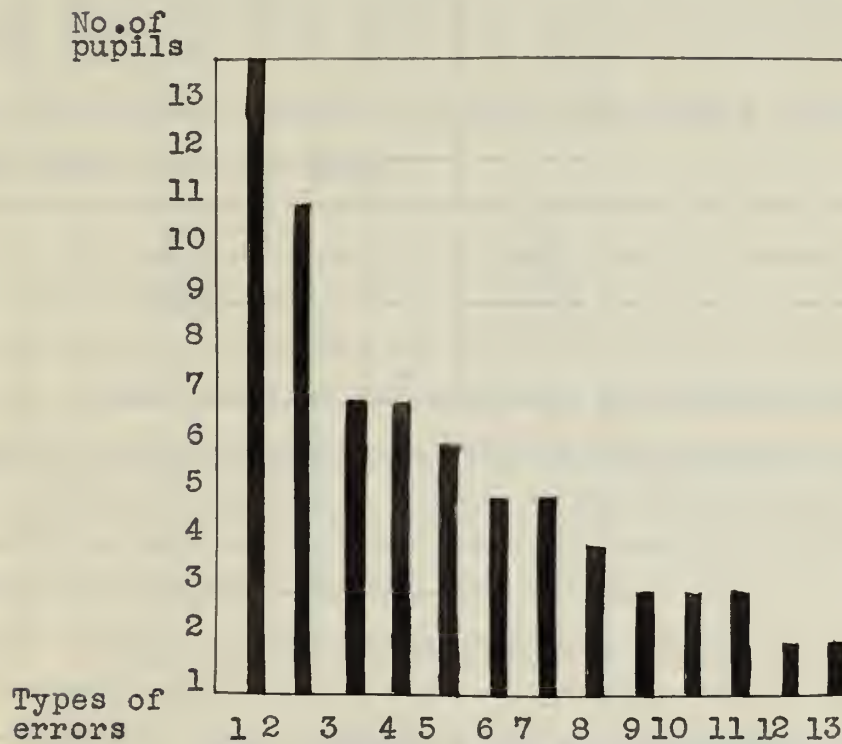
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Figure 6

Showing graphically the types of errors in the  
5 P Multiplication Test made by 38 Sixth Grade pupils

April 3, 1936



The 13 types of errors appearing at the bottom of the graph are numbered to correspond with the numbers of the types of errors in Table XV.



In Table XVI which follows on page 53 the distribution of the types of errors has been tabulated according to examples. The twenty-five examples are numbered alphabetically from a to y inclusive. In the fifth grade there were 155 errors while in the sixth grade there were 59 errors.

There were 7 errors in example "w." This example seemed to be the most difficult one in the test or rather the one on which the most errors were made. In example "r" there were 5 errors made.

(w)	(r)
784	54
<u>367</u>	<u>270</u>

Four of the seven errors in example w were in carrying in multiplication, and the five errors in example "r" were made in placing the last product in the wrong place on account of the zero in the multiplier.

Six pupils did example g incorrectly.

(g)	
\$5.90	The confusion lay in the zero in the
<u>10</u>	multiplicand and the zero in the multiplier.

In example "e" four errors were made in multiplying the digit times the zero. The other errors were fewer in number.



Sixth Grade Pupils, April 3, 1936

Ex.					
a					
b					
c					
d			1		1
e				4	4
f				2	3
g			6		6
h					
i				1	1
j		1			1
k	2			1	4
l	2			1	3
m					
n			1		1
o	1				2
p	1				1
q	1				1
r		5			5
s		1			1
t		1 1		1	3
u				1	1
v		1 1		1	3
w	4	1	1	1	7
x	2	1	1		6
y		2 1	1	1	5

Table XVIII is read as follows: example y contains 5 errors -- 2 "product in wrong place" (2); 1 "added instead of multiplying" (3) 1 "multiplied wrong-no carrying" (5); and 1 "omitted one product"(9)





The same type of instruction was followed as in the fifth grade. These pupils were tested on the 100 multiplication combinations and their weaknesses noted. Drill on the weak combinations followed. The steps according to difficulty were taken next and drilled upon. No drill was given on the combinations already mastered.

The Wilson Drill Service was followed as in the fifth grade work. (See pp.40-43) Each child moved at his own rate from one group of multiplication facts to the next as rapidly as perfect scores were secured. Most of the work in this grade was done individually and a record of the time spent in instruction and drill was kept. Flash card drill was given as in the fifth grade. (See pp.45 and 46)

The I.Q.'s of the pupils who received perfect scores ranged from 94 to 127 although the highest I.Q. in the class was 138. There were eleven pupils who received perfect scores on the first test given on April 3, 1936.

Five re-tests were given before every pupil received a perfect score. On the first re-test 3 pupils received perfect scores after 8 twenty-minute periods of remedial work; on the second re-test eleven pupils received perfect scores after eight additional twenty-minute periods of help in multiplication.

On the third re-test seven pupils received perfect scores after having eight more twenty-minute periods of instruction. On the fourth re-test two pupils received perfect

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scores after having five more twenty-minute periods of remedial work; and on the fifth re-test four pupils received perfect scores with three additional twenty-minute periods of instruction. 546 twenty-minute periods equalling 180 hours were used in remedial work with 27 pupils. All of the corrective work was done by the regular room teacher and the testing was done by the writer. The children enjoyed the work. It was fun to know that perfect scores were available for every one of them. Achievement brought the joy of accomplishment. The pupils no longer felt satisfied with a score of 96 when they realized that it was possible for them to receive 100 per cent.

A real desire for excellence could be seen in the daily work of these children. It was a great pleasure to work with them.

Tables with scores, time, and time allotted to remedial work follow herewith:









Table XVII continued

Showing the scores made and the time used in taking the first test and the total time allotted to remedial work with 38 sixth grade pupils between April 3, 1936 and June 19, 1936.

	Age	I.Q. N.I.T.	Apr. 3, 1936		Apr. 17		No. of 20 min. periods	May 14			May 28			June 11			June 19		
			Score	Time	Score	Time		Score	Time	No. of 20 min. periods	Score	Time	No. of 20 min. periods	Score	Time	No. of 20 min. periods	Score	Time	No. of 20 min. periods
31 Lawrence	12- 2	117	88	13	92	6	8	100	12	4									
32 Pauline	12- 0	106	88	17	96	7	8	96	20	8		23	8	100	27	5			
33 Teresa	11- 7	124	88	12	80	7	8	88	9	8	100	9							
34 Lois	11- 4	108	88	15	76	15	8	96	15	8	92	15		96	19		100	12	4
35 Beatrice	11- 6	128	84	11	100	10	8												
36 Phyllis	11- 2	115	84	14	88	12	8	96	9	8	92	9		96	10		100	10	
37 Paul	12-11	99	80	10	92	9	8	96	9	8	96	9		96	15		100	12	
38 Regina	10- 9	120	80	13	96	11	8	96	21	8	96	9		100	17		100	12	



Table XVIII

Showing a summary of the scores made and the time used in taking the first and the total time allotted to remedial work with 38 sixth grade pupils between April 3, 1936 and June 19, 1936.

		Age	I.Q. 1936				No. of 20 min. periods	No. of minutes			
			N.I.T.	Score	Time	Hours			and	Minutes)	
1	Jean Fr.	12- 8	: 102	: 100	: 20	:					
2	Lloyd	11- 5	: 107	: 100	: 14	:					
3	Andrew	11- 3	: 127	: 100	: 13	:					
4	Joseph	11- 9	: 120	: 100	: 19	:					
5	Mildred	11- 5	: 120	: 100	: 11	:					
6	Leona	12- 0	: 100	: 100	: 18	:					
7	Christina	11- 7	: 94	: 100	: 13	:					
8	Barbara	11- 9	: 109	: 100	: 13	:					
9	Daniel	11- 8	: 110	: 100	: 13	:					
10	Roland	13- 2	: 94	: 100	: 16	:					
11	Elaine	12- 0	: 115	: 100	: 13	:					
12	Maud	12- 1	: 110	: 96	: 17	: 16	: 320	:	5	-	20
13	Frank	12- 9	: 74	: 96	: 17	: 33	: 660	:	11	-	0
14	John Sch.	11- 6	: 116	: 96	: 18	: 16	: 320	:	5	-	20
15	Walter H.	11-10	: 116	: 96	: 13	: 24	: 480	:	8	-	0
16	Dorothy C.	10-10	: 100	: 96	: 14	: 24	: 480	:	8	-	0
17	Frances	12- 2	: 124	: 96	: 11	: 8	: 160	:	2	-	40
18	Virginia	10- 7	: 126	: 96	: 11	: 16	: 320	:	5	-	20
19	Gaitanna	12- 4	: 92	: 96	: 12	: 16	: 320	:	5	-	20
20	Kenneth	11- 4	: 108	: 96	: 19	: 16	: 320	:	5	-	20
21	Robert C.	11- 3	: 138	: 92	: 23	: 4	: 80	:	1	-	20
22	Walter L.	11- 3	: 112	: 92	: 10	: 24	: 480	:	8	-	0
23	Robert G.	11- 8	: 120	: 92	: 11	: 16	: 320	:	5	-	20
24	Raymond	11-10	: 134	: 92	: 11	: 24	: 480	:	8	-	0
25	Chandler	12- 0	: 105	: 92	: 27	: 16	: 320	:	5	-	20
26	Brenda	11- 8	: 120	: 92	: 23	: 8	: 160	:	2	-	40
27	Norma	11- 5	: 131	: 92	: 13	: 16	: 320	:	5	-	20
28	Herbert	11- 4	: 126	: 92	: 10	: 16	: 320	:	5	-	20
29	Mary Mc.	12- 0	: 118	: 92	: 27	: 24	: 480	:	8	-	0
30	Anthony	11-10	: 92	: 92	: 13	: 16	: 320	:	5	-	20
31	Lawrence	12- 2	: 117	: 88	: 13	: 24	: 480	:	8	-	0
32	Pauline	12- 0	: 106	: 88	: 17	: 29	: 580	:	9	-	40
33	Teresa	11- 7	: 124	: 88	: 12	: 24	: 480	:	8	-	0
34	Lois	11- 4	: 108	: 88	: 15	: 33	: 660	:	1	-	0
35	Beatrice	11- 6	: 120	: 84	: 11	: 8	: 160	:	2	-	40
36	Phyllis	11- 2	: 115	: 84	: 14	: 33	: 660	:	11	-	0
37	Paul	12-11	: 99	: 80	: 10	: 33	: 660	:	11	-	0
38	Regina	10- 9	: 120	: 80	: 13	: 29	: 580	:	9	-	40
Average					20	400		6	-	40	

An average of 20 twenty-minute periods or 6 hr. and 40 minutes was given to each sixth grade pupil for remedial work.





### Conclusions

1. The study justifies the challenge and tentative conclusion that every pupil of normal intelligence in the fifth and sixth grades can secure perfect mastery in multiplication if the teaching is satisfactory.

2. Some pupils with I.Q.'s lower than 90 can secure perfect mastery in multiplication. Sometimes the I.Q. as named by the N.I.T. is lower than 90 because of language handicap yet the pupil is able to learn and retain anything that he is able to understand. Some pupils with an I.Q. lower than 90 are able to learn and retain subject matter that is a tool subject taught on a drill basis.

3. The counting habit or "saying the tables" can be broken up by teaching the multiplication combinations in unrelated groups. The tables appear to be a definite handicap.

4. Automatic response to the multiplication combinations and the higher decade facts in addition used in carrying will eliminate counting and "saying the tables."

5. Automatic response to the multiplication combinations and the higher decade facts used in carrying will develop accuracy and result in perfect mastery in multiplication.

6. The time needed to bring a fifth grade pupil to perfect scores in multiplication is shown in Tables XI and XII. In general a child with an I.Q. of 107, an initial score of 84, can be brought to a perfect score in 48 twenty-minute



periods of corrective work. This is a statement based on medians.

7. The Time needed to bring a sixth grade pupil to perfect scores in multiplication is shown in Tables XVII and XVIII. In general a child with an I.Q. of 112, an initial score of 96 can be brought to a perfect score in 20 twenty-minute periods of corrective work. This is a statement based on medians.

8. The question of maintenance should receive attention. Re-tests should be given to the pupils receiving perfect scores to see if they are able to reach perfect mastery and retain it. This phase of the work is not considered in the present study.

9. The author wishes to add that the above work proceeded with understanding and motivation. It was not meaningless and mechanical. The children readily conceived the goal, accepted it as reasonable, and worked enthusiastically for its attainment.





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CHAPTER III

The first part of the chapter is devoted to a discussion of the general principles of the theory of the structure of the atom. It is shown that the structure of the atom is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are based on the principles of wave mechanics.

The second part of the chapter is devoted to a discussion of the general principles of the theory of the structure of the molecule. It is shown that the structure of the molecule is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are based on the principles of wave mechanics.

The third part of the chapter is devoted to a discussion of the general principles of the theory of the structure of the crystal. It is shown that the structure of the crystal is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are based on the principles of wave mechanics.

The fourth part of the chapter is devoted to a discussion of the general principles of the theory of the structure of the liquid. It is shown that the structure of the liquid is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are based on the principles of wave mechanics.

The fifth part of the chapter is devoted to a discussion of the general principles of the theory of the structure of the gas. It is shown that the structure of the gas is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are based on the principles of wave mechanics.

The sixth part of the chapter is devoted to a discussion of the general principles of the theory of the structure of the plasma. It is shown that the structure of the plasma is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are based on the principles of wave mechanics.

The seventh part of the chapter is devoted to a discussion of the general principles of the theory of the structure of the solid. It is shown that the structure of the solid is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are based on the principles of wave mechanics.

The eighth part of the chapter is devoted to a discussion of the general principles of the theory of the structure of the liquid crystal. It is shown that the structure of the liquid crystal is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are based on the principles of wave mechanics.

The ninth part of the chapter is devoted to a discussion of the general principles of the theory of the structure of the polymer. It is shown that the structure of the polymer is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are based on the principles of wave mechanics.

The tenth part of the chapter is devoted to a discussion of the general principles of the theory of the structure of the composite material. It is shown that the structure of the composite material is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are based on the principles of wave mechanics.

The eleventh part of the chapter is devoted to a discussion of the general principles of the theory of the structure of the nanomaterial. It is shown that the structure of the nanomaterial is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are based on the principles of wave mechanics.

The twelfth part of the chapter is devoted to a discussion of the general principles of the theory of the structure of the biomaterial. It is shown that the structure of the biomaterial is determined by the laws of quantum mechanics, and that the laws of quantum mechanics are based on the principles of wave mechanics.

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1. The first part of the paper is devoted to a general discussion of the problem of the existence of solutions of the system of equations

which are satisfied by the functions  $u_i(x, y, z)$  and  $v_i(x, y, z)$  in the domain  $D$ .

2. In the second part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be continuous in the domain  $D$ .

3. In the third part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be differentiable in the domain  $D$ .

4. In the fourth part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be twice differentiable in the domain  $D$ .

5. In the fifth part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be three times differentiable in the domain  $D$ .

6. In the sixth part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be four times differentiable in the domain  $D$ .

7. In the seventh part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be five times differentiable in the domain  $D$ .

8. In the eighth part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be six times differentiable in the domain  $D$ .

9. In the ninth part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be seven times differentiable in the domain  $D$ .

10. In the tenth part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be eight times differentiable in the domain  $D$ .

11. In the eleventh part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be nine times differentiable in the domain  $D$ .

12. In the twelfth part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be ten times differentiable in the domain  $D$ .

13. In the thirteenth part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be eleven times differentiable in the domain  $D$ .

14. In the fourteenth part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be twelve times differentiable in the domain  $D$ .

15. In the fifteenth part we shall consider the case when the functions  $u_i$  and  $v_i$  are assumed to be thirteen times differentiable in the domain  $D$ .



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1. The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is of great importance in the theory of differential equations.

2. In the second part, we consider the case of a linear differential equation. It is shown that the problem is solvable in this case.

3. In the third part, we consider the case of a nonlinear differential equation. It is shown that the problem is solvable in this case.

4. In the fourth part, we consider the case of a system of differential equations. It is shown that the problem is solvable in this case.

5. In the fifth part, we consider the case of a partial differential equation. It is shown that the problem is solvable in this case.

6. In the sixth part, we consider the case of a boundary value problem. It is shown that the problem is solvable in this case.

7. In the seventh part, we consider the case of an initial value problem. It is shown that the problem is solvable in this case.

8. In the eighth part, we consider the case of a mixed boundary value problem. It is shown that the problem is solvable in this case.

9. In the ninth part, we consider the case of a problem with variable coefficients. It is shown that the problem is solvable in this case.

10. In the tenth part, we consider the case of a problem with discontinuous coefficients. It is shown that the problem is solvable in this case.

11. In the eleventh part, we consider the case of a problem with nonlocal conditions. It is shown that the problem is solvable in this case.

12. In the twelfth part, we consider the case of a problem with nonlocal conditions. It is shown that the problem is solvable in this case.

13. In the thirteenth part, we consider the case of a problem with nonlocal conditions. It is shown that the problem is solvable in this case.

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Primary Grades." Childhood Education.  
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1. The first part of the report deals with the general situation of the country. It is a very interesting and informative study of the country's development. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is easy to read. It is a valuable contribution to the study of the country's development.

2. The second part of the report deals with the economic situation of the country. It is a very interesting and informative study of the country's economic development. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is easy to read. It is a valuable contribution to the study of the country's economic development.

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4. The fourth part of the report deals with the political situation of the country. It is a very interesting and informative study of the country's political development. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is easy to read. It is a valuable contribution to the study of the country's political development.

5. The fifth part of the report deals with the cultural situation of the country. It is a very interesting and informative study of the country's cultural development. The author has done a great deal of research and has gathered a wealth of material. The report is well written and is easy to read. It is a valuable contribution to the study of the country's cultural development.



## Wilson 5 P Multiplication Test

Multiply

$$\begin{array}{r} \text{(a)} \quad 8 \quad 7 \quad 4 \quad 2 \quad 4 \quad 7 \quad 3 \quad 4 \quad 8 \quad 6 \\ \quad \quad \underline{6} \quad \underline{3} \quad \underline{9} \quad \underline{7} \quad \underline{1} \quad \underline{5} \quad \underline{3} \quad \underline{2} \quad \underline{4} \quad \underline{7} \end{array}$$

$$\begin{array}{r} \text{(b)} \\ \$3.65 \\ \quad \underline{6} \end{array}$$

$$\begin{array}{r} \text{(c)} \\ 501 \\ \quad \underline{6} \end{array}$$

$$\begin{array}{r} \text{(d)} \\ \$8.05 \\ \quad \underline{7} \end{array}$$

$$\begin{array}{r} \text{(e)} \quad 7 \quad 4 \quad 0 \quad 4 \quad 0 \quad 3 \quad 5 \quad 4 \quad 8 \quad 0 \\ \quad \quad \underline{0} \quad \underline{5} \quad \underline{8} \quad \underline{4} \quad \underline{1} \quad \underline{4} \quad \underline{2} \quad \underline{0} \quad \underline{9} \quad \underline{5} \end{array}$$

$$\begin{array}{r} \text{(f)} \\ \$7.40 \\ \quad \underline{6} \end{array}$$

$$\begin{array}{r} \text{(g)} \\ \$5.90 \\ \quad \underline{10} \end{array}$$

$$\begin{array}{r} \text{(h)} \\ \$700.95 \\ \quad \underline{4} \end{array}$$

$$\begin{array}{r} \text{(i)} \\ 70 \\ \underline{17} \end{array}$$

$$\begin{array}{r} \text{(j)} \\ 362 \\ \underline{21} \end{array}$$

$$\begin{array}{r} \text{(k)} \\ 95 \\ \underline{47} \end{array}$$

$$\begin{array}{r} \text{(l)} \\ 92 \\ \underline{56} \end{array}$$

$$\begin{array}{r} \text{(m)} \\ 93 \\ \underline{89} \end{array}$$

$$\begin{array}{r} \text{(n)} \\ \$7.30 \\ \quad \underline{29} \end{array}$$

$$\begin{array}{r} \text{(o)} \\ 896 \\ \underline{83} \end{array}$$

$$\begin{array}{r} \text{(p)} \\ 693 \\ \underline{600} \end{array}$$

$$\begin{array}{r} \text{(q)} \\ 445 \\ \underline{308} \end{array}$$

$$\begin{array}{r} \text{(r)} \\ 54 \\ \underline{270} \end{array}$$

$$\begin{array}{r} \text{(s)} \\ 7081 \\ \underline{509} \end{array}$$

$$\begin{array}{r} \text{(t)} \\ \$680 \\ \underline{120} \end{array}$$

$$\begin{array}{r} \text{(u)} \\ 915 \\ \underline{504} \end{array}$$

$$\begin{array}{r} \text{(v)} \\ 506 \\ \underline{129} \end{array}$$

$$\begin{array}{r} \text{(w)} \\ 784 \\ \underline{367} \end{array}$$

$$\begin{array}{r} \text{(x)} \\ 8302 \\ \underline{805} \end{array}$$

$$\begin{array}{r} \text{(y)} \\ 842 \\ \underline{2100} \end{array}$$

# Mathematical Induction

Example 1

$1$	$2$	$3$	$4$	$5$	$6$	$7$	$8$	$9$	$10$
$1$	$1$	$1$	$1$	$1$	$1$	$1$	$1$	$1$	$1$

(a)  $1$   $2$   $3$   $4$   $5$   $6$   $7$   $8$   $9$   $10$

(b)  $1$   $2$   $3$   $4$   $5$   $6$   $7$   $8$   $9$   $10$

(c)  $1$   $2$   $3$   $4$   $5$   $6$   $7$   $8$   $9$   $10$

(d)  $1$   $2$   $3$   $4$   $5$   $6$   $7$   $8$   $9$   $10$

Represents the 100 Multiplication Combinations, used in

FLASH CARD DRILL

0 X 7	7 X 0	3 X 3	2 X 2
5 X 7	7 X 5	2 X 1	1 X 2
2 X 5	5 X 2	5 X 0	0 X 5
9 X 5	5 X 9	9 X 7	7 X 9
1 X 7	7 X 1	2 X 6	6 X 2
4 X 4	0 X 0	7 X 8	8 X 7
4 X 5	5 X 4	1 X 8	8 X 1
8 X 2	2 X 8	0 X 4	4 X 0
6 X 9	9 X 6	4 X 3	3 X 4
9 X 1	1 X 9	1 X 3	3 X 1
3 X 0	0 X 3	7 X 7	9 X 9
1 X 6	6 X 1	4 X 9	9 X 8
6 X 3	3 X 6	2 X 0	0 X 2
4 X 8	8 X 4	2 X 7	7 X 2
1 X 4	4 X 1	4 X 9	9 X 4
6 X 0	0 X 6	4 X 7	7 X 4
8 X 3	3 X 8	4 X 2	2 X 4
6 X 6	5 X 5	0 X 8	8 X 0
2 X 3	3 X 2	6 X 4	4 X 6
3 X 5	5 X 3	7 X 6	6 X 7
0 X 9	9 X 0	8 X 6	6 X 8
2 X 9	9 X 2	5 X 1	1 X 5
5 X 8	8 X 5	0 X 1	1 X 0
6 X 5	5 X 6	7 X 3	3 X 7
8 X 8	1 X 1	3 X 9	9 X 3



Multiplication Combinations plus the carrying figure used in

FLASH CARD DRILL

$6 \times 6 \rightarrow 3$

$6 \times 3 \rightarrow 3$

$6 \times 7 \rightarrow 2$

$4 \times 9 \rightarrow 2$

$2 \times 3 \rightarrow 1$

$7 \times 9 \rightarrow 3$

$4 \times 9 \rightarrow 3$

$6 \times 9 \rightarrow 1$

$5 \times 9 \rightarrow 1$

$9 \times 9 \rightarrow 2$

$8 \times 9 \rightarrow 2$

$9 \times 7 \rightarrow 2$

$3 \times 9 \rightarrow 1$

$3 \times 8 \rightarrow 2$

$8 \times 9 \rightarrow 4$

$8 \times 8 \rightarrow 7$

$8 \times 4 \rightarrow 4$

$8 \times 4 \rightarrow 3$

$3 \times 4 \rightarrow 1$

$7 \times 5 \rightarrow 2$

$2 \times 6 \rightarrow 1$

$7 \times 8 \rightarrow 2$

$7 \times 7 \rightarrow 5$

$6 \times 8 \rightarrow 2$

$6 \times 7 \rightarrow 5$

$3 \times 8 \rightarrow 1$

$3 \times 7 \rightarrow 2$

$5 \times 8 \rightarrow 1$

$8 \times 8 \rightarrow 2$

$7 \times 9 \rightarrow 2$

$9 \times 5 \rightarrow 2$

$9 \times 7 \rightarrow 3$

$8 \times 3 \rightarrow 2$

$4 \times 3 \rightarrow 1$

$9 \times 4 \rightarrow 3$

$8 \times 7 \rightarrow 1$

$9 \times 6 \rightarrow 1$

$7 \times 6 \rightarrow 5$

$3 \times 6 \rightarrow 2$

$9 \times 7 \rightarrow 4$

$4 \times 8 \rightarrow 3$

$6 \times 2 \rightarrow 1$

$7 \times 6 \rightarrow 2$

$4 \times 3 \rightarrow 2$

$7 \times 3 \rightarrow 2$

$8 \times 4 \rightarrow 5$

$6 \times 5 \rightarrow 2$

$4 \times 6 \rightarrow 3$

$5 \times 7 \rightarrow 4$

$7 \times 9 \rightarrow 4$

$4 \times 7 \rightarrow 3$

$6 \times 4 \rightarrow 5$

$8 \times 3 \rightarrow 6$

$5 \times 6 \rightarrow 3$

$4 \times 4 \rightarrow 3$

$5 \times 9 \rightarrow 4$

$6 \times 9 \rightarrow 4$

$3 \times 4 \rightarrow 2$

$8 \times 4 \rightarrow 7$

$6 \times 9 \rightarrow 2$

$6 \times 5 \rightarrow 5$

$4 \times 8 \rightarrow 1$

$8 \times 7 \rightarrow 6$

$9 \times 3 \rightarrow 3$

$4 \times 3 \rightarrow 3$

$7 \times 6 \rightarrow 3$

$8 \times 7 \rightarrow 2$

$4 \times 8 \rightarrow 2$

$5 \times 6 \rightarrow 4$









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